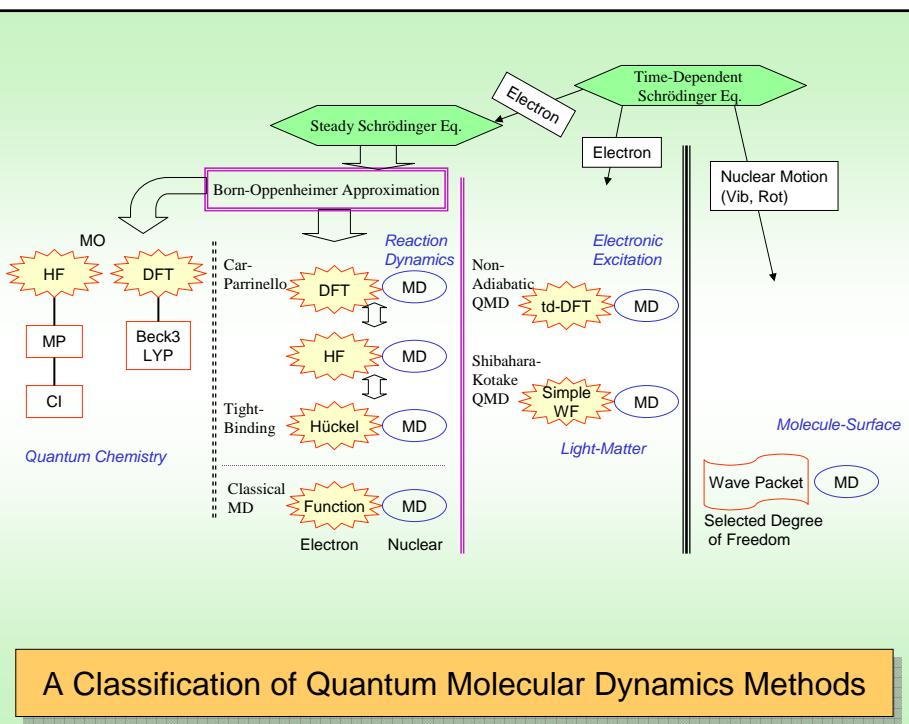
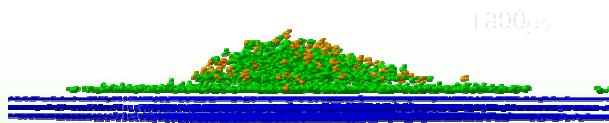


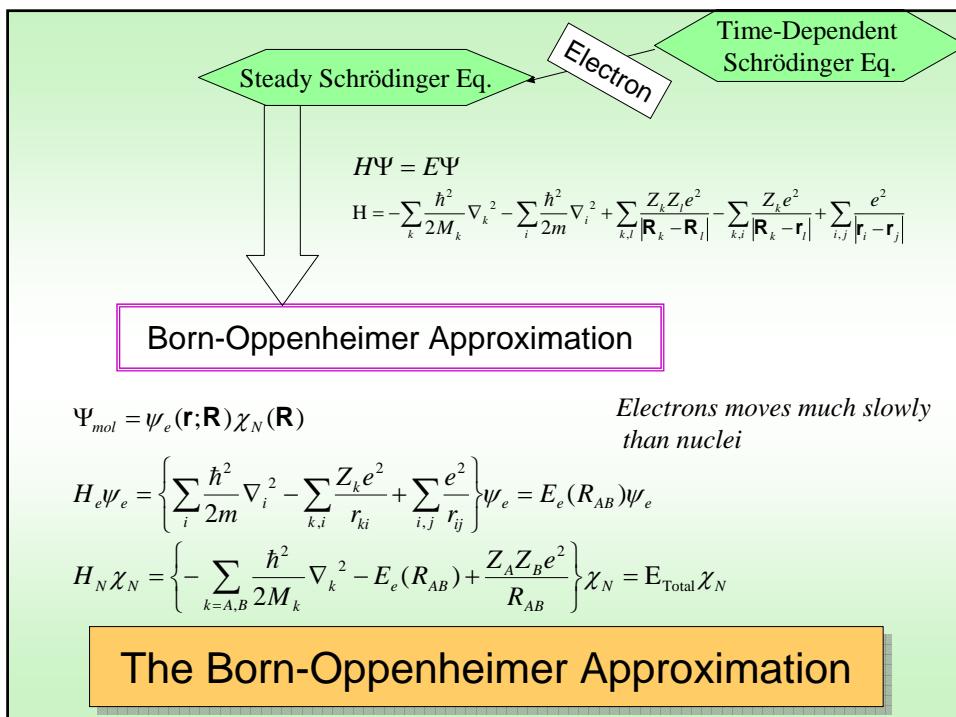
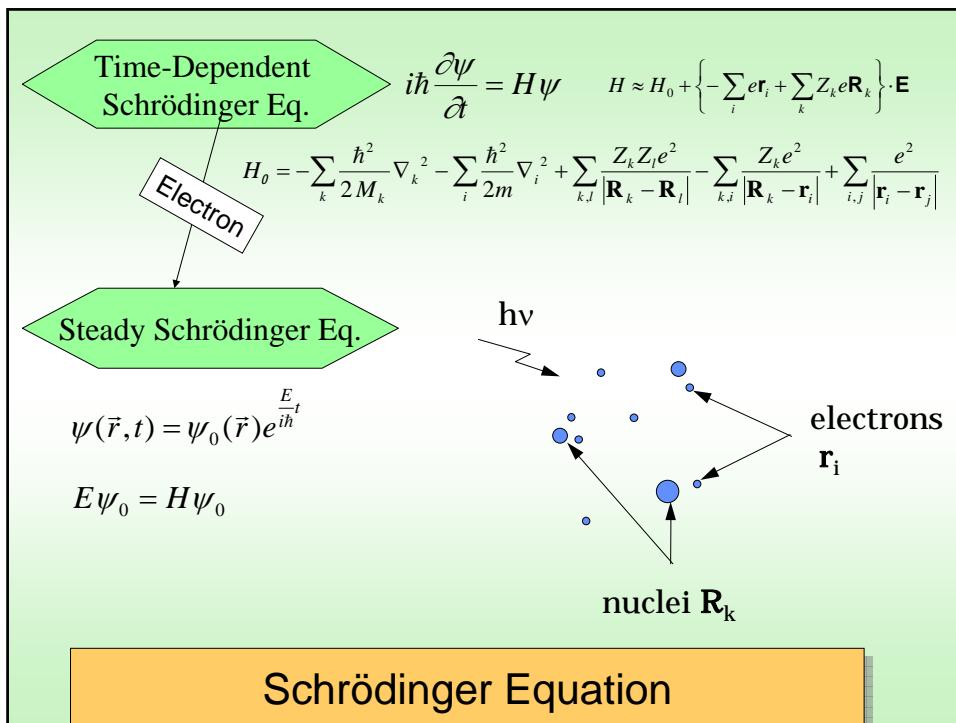
熱工学の新領域に関するクライミング・セミナー @ 仙台 2004.11.12

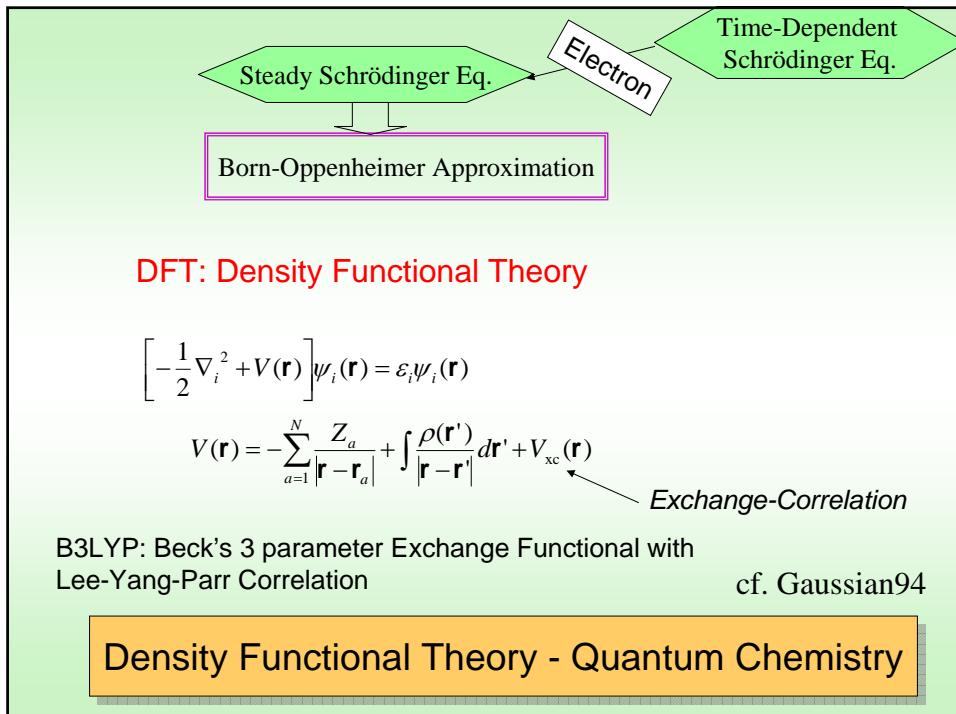
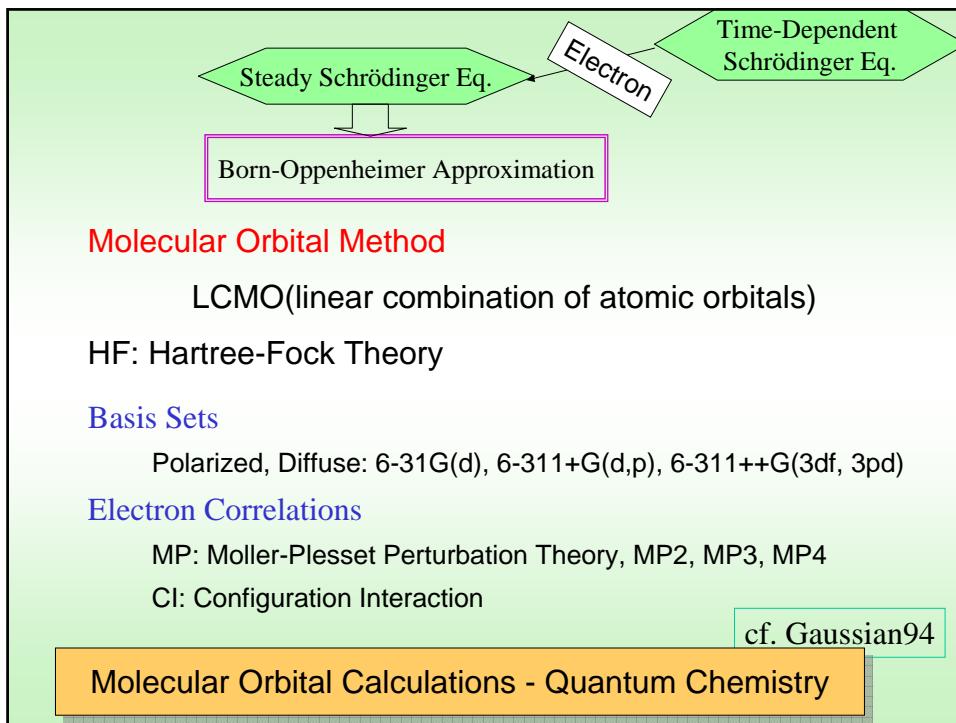
## マイクロからナノへの熱工学 (1) 分子動力学の基礎と応用

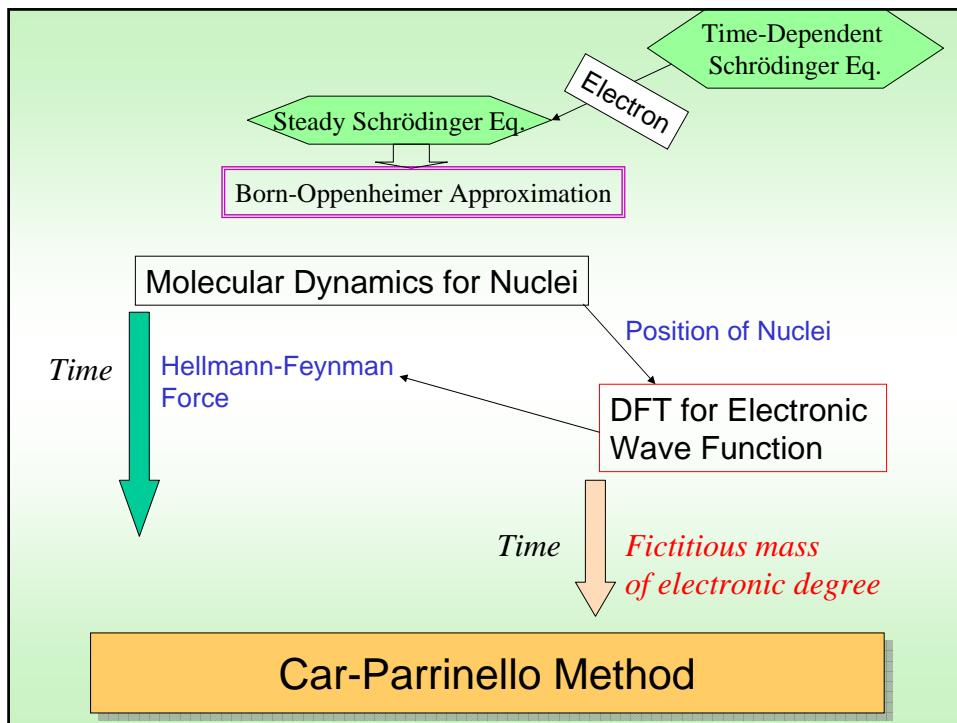
丸山 茂夫  
東京大学大学院 工学系研究科 機械工学専攻

<http://www.photon.t.u-tokyo.ac.jp>









分子動力学法の基礎

Newton's equation of motion

$$m_i \frac{d^2 \mathbf{r}_i}{dt^2} = \mathbf{F}_i = -\nabla \Phi$$

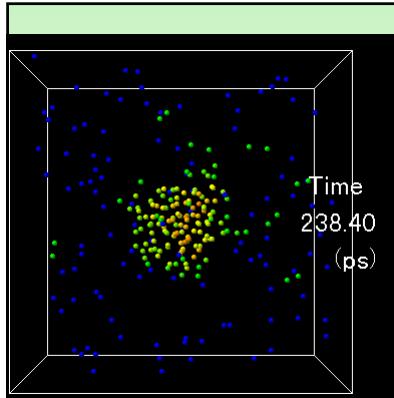
Approximation of Schrödinger Equation

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

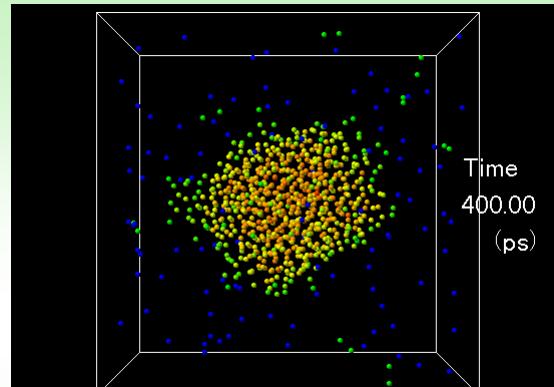
Pair Potential Approximation

$$\Phi = \sum_i \sum_{j>i} \phi(r_{ij})$$

運動方程式とポテンシャル

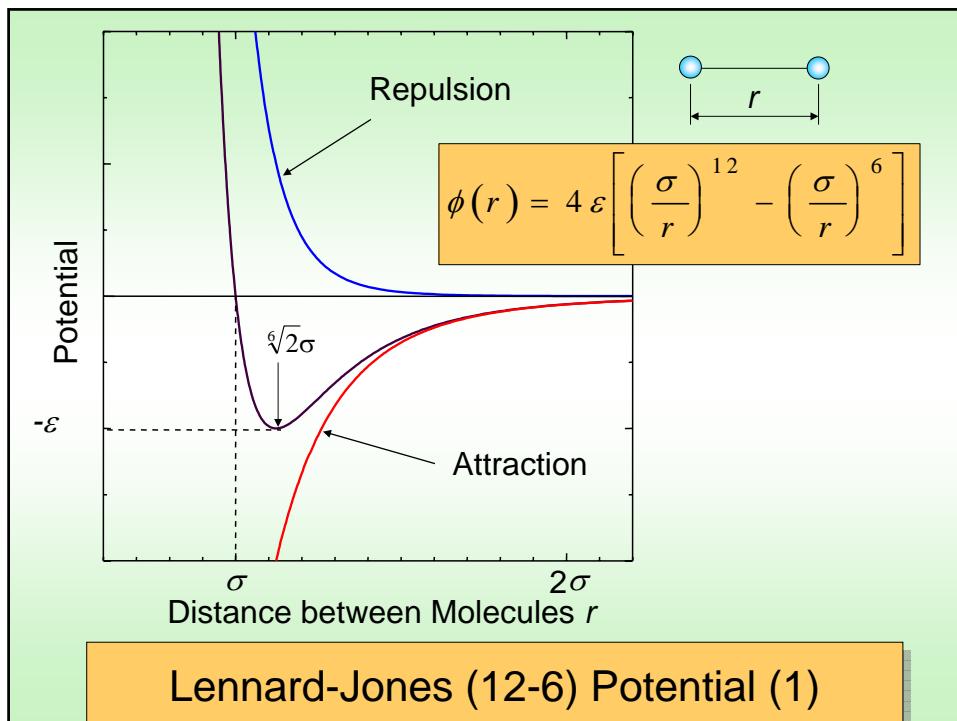


Only 256 molecules



864 molecules

Small Droplets



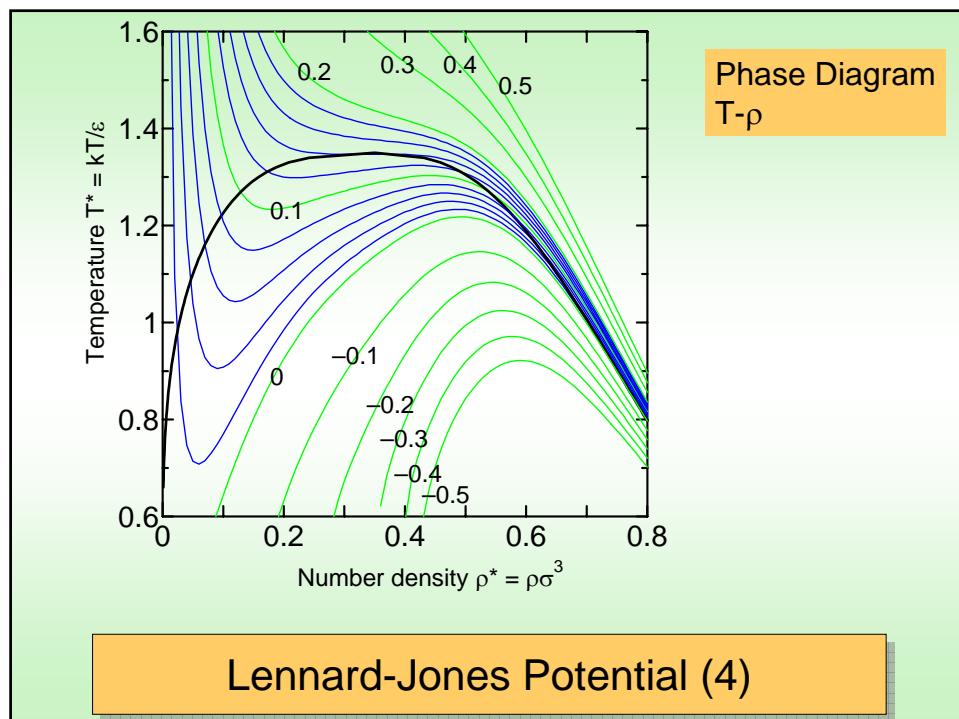
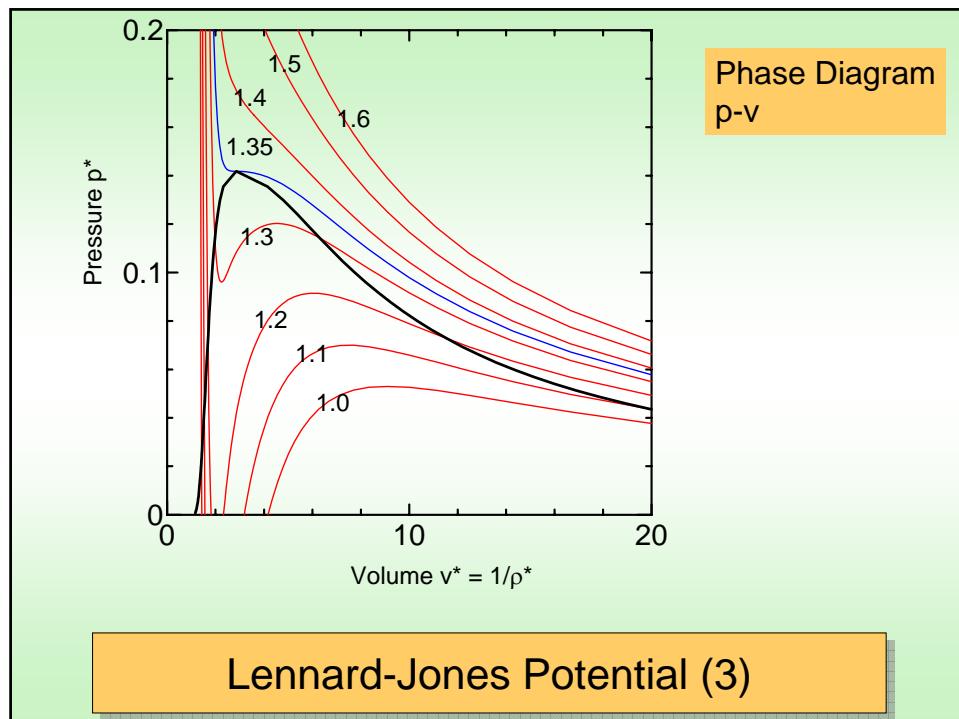
### Lennard-Jones Potential (2)

Parameters for inert molecules

	$\sigma$ [nm]	$\varepsilon$ [J]	$\varepsilon/k_B$ [K]
Ne	0.274	$0.50 \times 10^{-21}$	36.2
Ar	0.340	$1.67 \times 10^{-21}$	121
Kr	0.365	$2.25 \times 10^{-21}$	163
Xe	0.398	$3.20 \times 10^{-21}$	232

Non-dimensional Form for L-J System

Property	Reduced Form
Length	$r^* = r/\sigma$
Time	$t^* = t/\tau = t(\varepsilon/m\sigma^2)^{1/2}$
Temperature	$T^* = k_B T/\varepsilon$
Force	$f^* = f\sigma/\varepsilon$
Energy	$\phi^* = \phi/\varepsilon$
Pressure	$P^* = P\sigma^3/\varepsilon$
Number density	$N^* = N\sigma^3$
Density	$\rho^* = \sigma^3 \rho/m$
Surface Tension	$\gamma^* = \gamma\sigma^2/\varepsilon$



## Empirical Relations

Thermodynamics State Value for Lennard-Jones Fluid

Non-Dimensional Temperature  $T^* = 1.2$        $t^* = \pi T^*/\epsilon$

Non-dimensional number density  $\rho^* = 0.7$        $\rho = \rho^* \sigma^3 \times (4\pi/\sigma^3)^{1/3}$

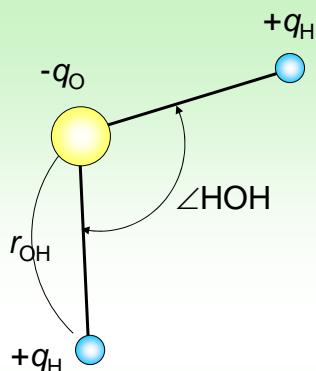
Ree	Nicolas et al.
$p^* = \rho \sigma^3 / \epsilon$	0.63425540
$pV/p(\epsilon T)$	0.75506986
$A^* = A\sigma/(NkT)$	-2.0848513
$A^* = A\sigma/(NkT)$	-1.7333261
$\mu^* = \mu\sigma/\epsilon$	-2.3787721
$\mu^* = \mu\sigma/\epsilon$	-1.9823101
$U^* = U/(NkT)$	-4.7222339
$U^* = U/(NkT)$	-3.9351949
$E^* = E/(NkT)$	-2.9222309
$E^* = E/(NkT)$	-2.4351949
$S^* = S/(Nk)$	-2.1978160
	-2.2293064
	Residual Entropy / atom

Temperature Density

Pressure  
 Helmholtz Free Energy  
 Gibbs Free Energy  
 Potential Energy  
 Internal Energy  
 Entropy

Ree correlation  
 and Nicolas et al. correlation

<http://www.photon.t.u-tokyo.ac.jp/~maruyama/ljphase/ljphase.html>

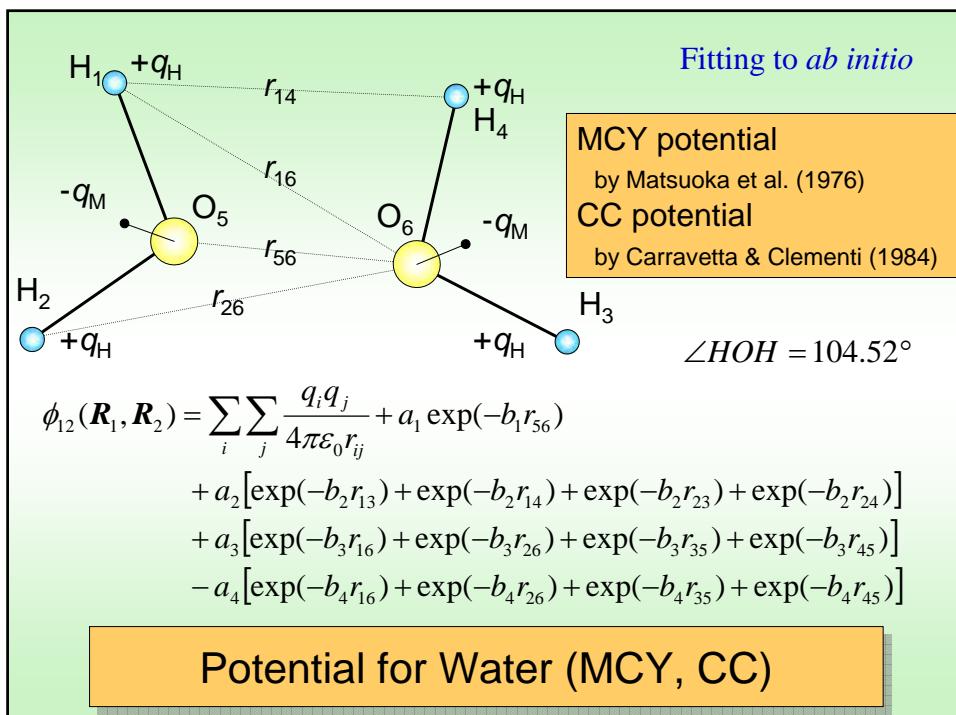
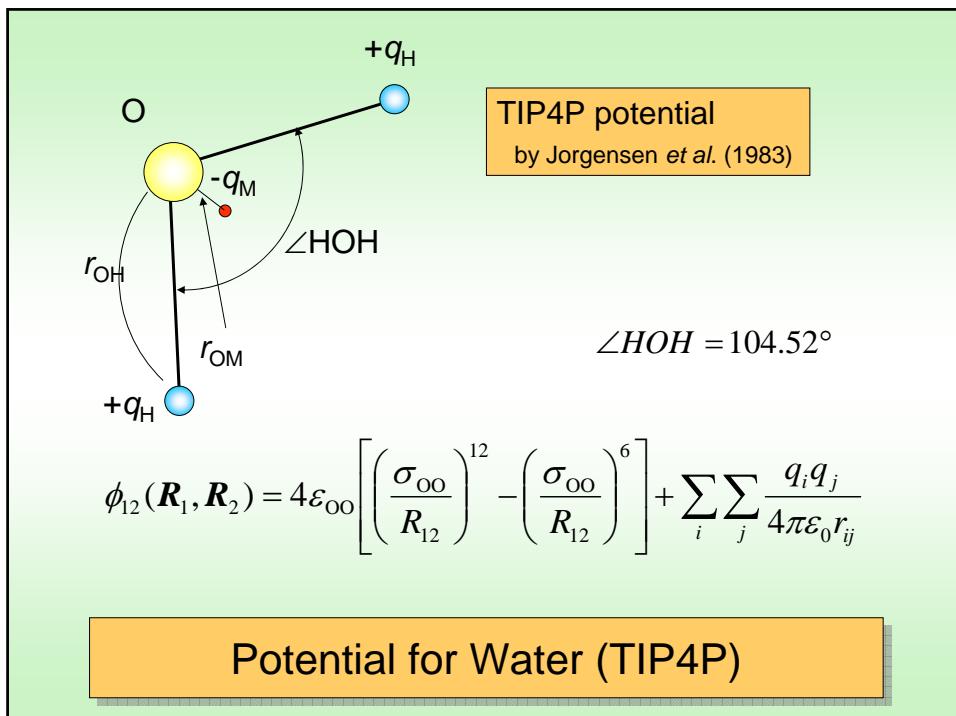


SPC potential  
 by Berendsen et al. (1981)  
 SPC/E potential  
 by Berendsen et al. (1987)

$$\angle HOH = 2 \cos^{-1}(1/\sqrt{3}) = 109.47^\circ$$

$$\phi_{12}(\mathbf{R}_1, \mathbf{R}_2) = 4\epsilon_{OO} \left[ \left( \frac{\sigma_{OO}}{R_{12}} \right)^{12} - \left( \frac{\sigma_{OO}}{R_{12}} \right)^6 \right] + \sum_i \sum_j \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

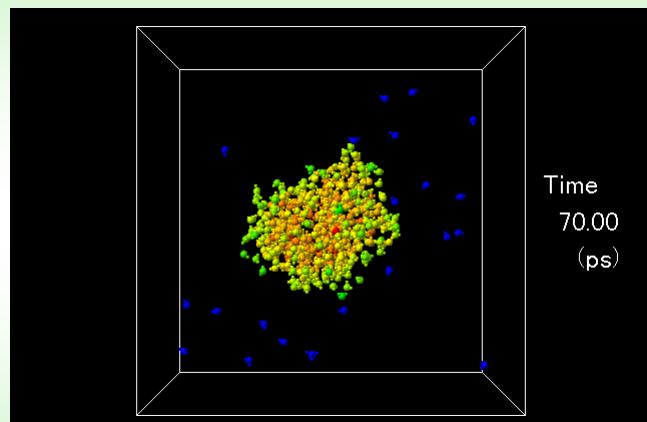
Potential for Water (SPC, SPC/E)



## Potential for Water (Comparison)

		ST2	SPC/E	TIP4P	CC
$r_{\text{OH}}$	[nm]	0.100	0.100	0.095 72	0.095 72
$\angle \text{HOH}$	[°]	109.47	109.47	104.52	104.52
$\sigma_{\text{OO}}$	[nm]	0.310	0.316 6	0.315 4	N/A
$\varepsilon_{\text{OO}} \times 10^{-21}$	[J]	0.526 05	1.079 7	1.077 2	N/A
$r_{\text{OM}}$	[nm]	0.08	0	0.015	0.024 994
$q_{\text{H}}^{\text{a}}$	[C]	0.235 7 e	0.423 8 e	0.52 e	0.185 59 e
$q_{\text{M}}$	[C]	-0.235 7 e	-0.847 6 e	-1.04 e	-0.371 18 e

<sup>a</sup>Charge of electron  $e = 1.60219 \times 10^{-19} \text{ C}$



## Droplet of Water

Tersoff (1988, 1989), Brenner (1990)

$$\Phi = \sum_i \sum_{j(i < j)} f_C(r_{ij}) \{ V_R(r_{ij}) - b_{ij}^* V_A(r_{ij}) \}$$

$$V_R(r) = f_C(r) \frac{D_e}{S-1} \exp\left\{-\beta\sqrt{2S}(r-R_e)\right\}$$

$$V_A(r) = f_C(r) \frac{D_e S}{S-1} \exp\left\{-\beta\sqrt{2/S}(r-R_e)\right\}$$

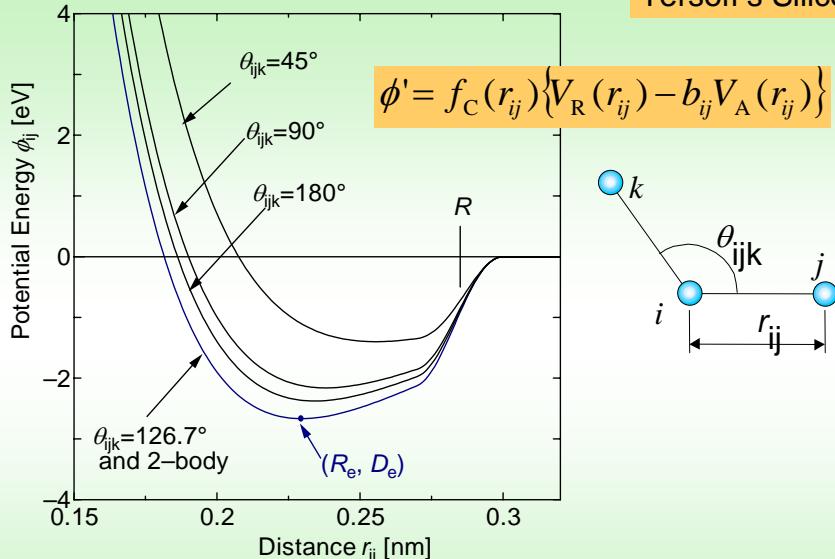
$$f_C(r) = \begin{cases} 1 & (r < R-D) \\ \frac{1}{2} - \frac{1}{2} \sin\left[\frac{\pi}{2}(r-R)/D\right] & (R-D < r < R+D) \\ 0 & (r > R+D) \end{cases}$$

$$b_{ij}^* = \frac{b_{ij} + b_{ji}}{2} \quad b_{ij} = \left(1 + a^n \left\{ \sum_{k(\neq i, j)} f_C(r_{ik}) g(\theta_{ijk}) \right\}^n\right)^{-\delta}$$

$$g(\theta) = 1 + \frac{c^2}{d^2} - \frac{c^2}{d^2 + (h - \cos \theta)^2}$$

Potential for Covalent System (C, Si)

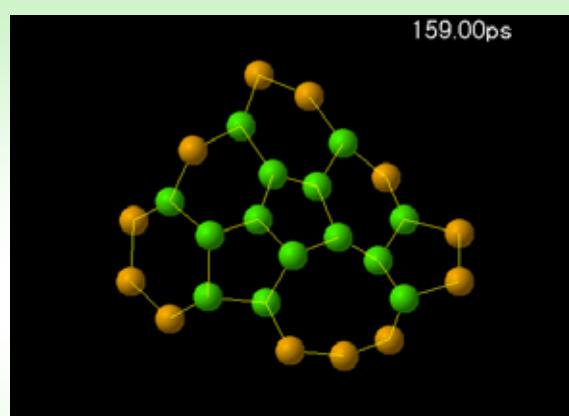
Tersoff's Silicon



Potential for Covalent System (C, Si)

	Tersoff (Si)	Tersoff (C)	Brenner (C)
$D_e$ [eV]	2.6660	5.1644	6.325
$R_e$ [nm]	0.2295	0.1447	0.1315
$S$	1.4316	1.5769	1.29
$\beta$ [nm $^{-1}$ ]	14.656	19.640	1.5
$A$	$1.1000 \times 10^{-6}$	$1.5724 \times 10^{-7}$	$1.1304 \times 10^{-2}$
$N$	$7.8734 \times 10^{-1}$	$7.2751 \times 10^{-1}$	1
$\delta$	$1/(2n)$	$1/(2n)$	0.80469
$c$	$1.0039 \times 10^5$	$3.8049 \times 10^4$	19
$d$	$1.6217 \times 10^1$	4.384	2.5
$h$	$-5.9825 \times 10^{-1}$	$-5.7058 \times 10^{-1}$	-1
$R$ [nm]	0.285	0.195	0.185
$D$ [nm]	0.015	0.015	0.015

### Potential for Covalent System (C, Si)



Example: Brenner Carbon (modified)

### Potential for Covalent System (C, Si)

### Verlet's Method

$$\mathbf{r}_i(t + \Delta t) = 2\mathbf{r}_i(t) - \mathbf{r}_i(t - \Delta t) + (\Delta t)^2 \mathbf{F}_i(t)/m_i$$

$$\mathbf{v}_i(t) = \{\mathbf{r}_i(t + \Delta t) - \mathbf{r}_i(t - \Delta t)\}/2\Delta t$$

### Leap Flog Method (Modified Verlet)

$$\mathbf{v}_i\left(t + \frac{\Delta t}{2}\right) = \mathbf{v}_i\left(t - \frac{\Delta t}{2}\right) + \Delta t \frac{\mathbf{F}_i(t)}{m_i}$$

$$\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \Delta t \mathbf{v}_i\left(t + \frac{\Delta t}{2}\right)$$

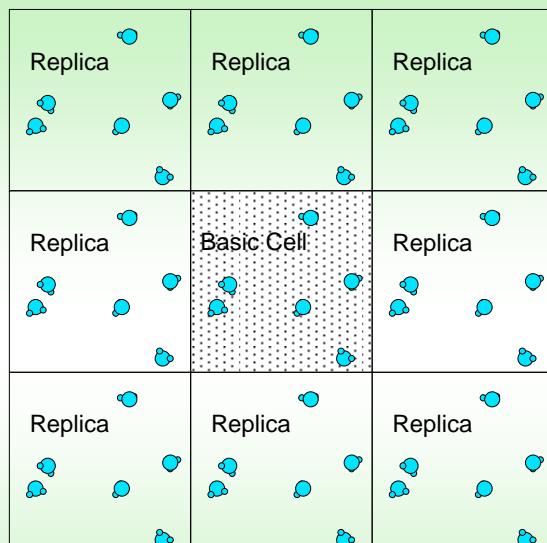
Order of  $\Delta t$

0.005  $\tau$  or 10 fs with argon

0.5 fs for covalent Carbon

Gear's predictor-corrector method  
is also sometimes used

## Integration of Newton's Equation



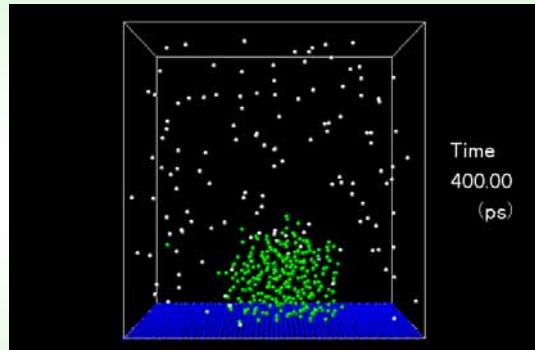
Potential must be  
Cut-Off at  $L/2$

Ewald sum method  
for Coulomb Term

## Boundary Condition (Periodic)

Example

Mirror Boundary  
=Simple Reflection

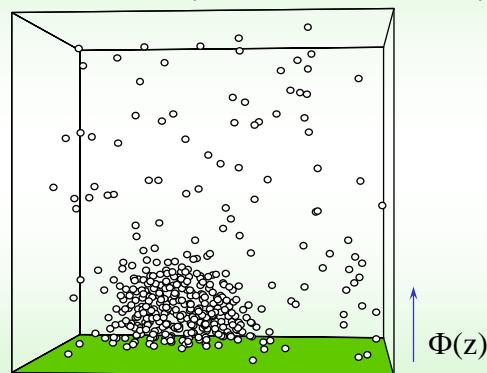


Periodic Boundary

Boundary Condition (Gas)

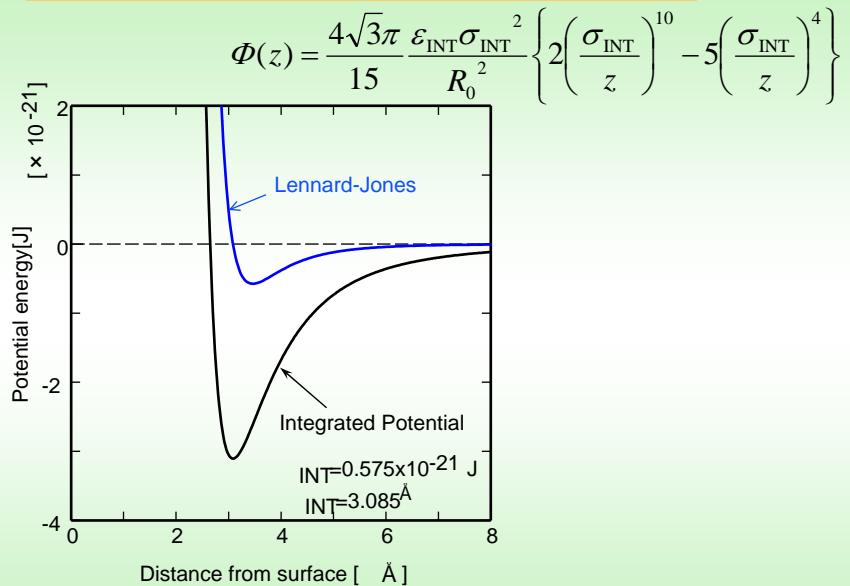
One-Dimensional Function by Bulk Integration

$$\Phi(z) = \frac{2\pi}{45} \frac{\rho_s}{m_s} \epsilon_{\text{INT}} \sigma_{\text{INT}}^3 \left\{ 2 \left( \frac{\sigma_{\text{INT}}}{z} \right)^9 - 15 \left( \frac{\sigma_{\text{INT}}}{z} \right)^3 \right\}$$

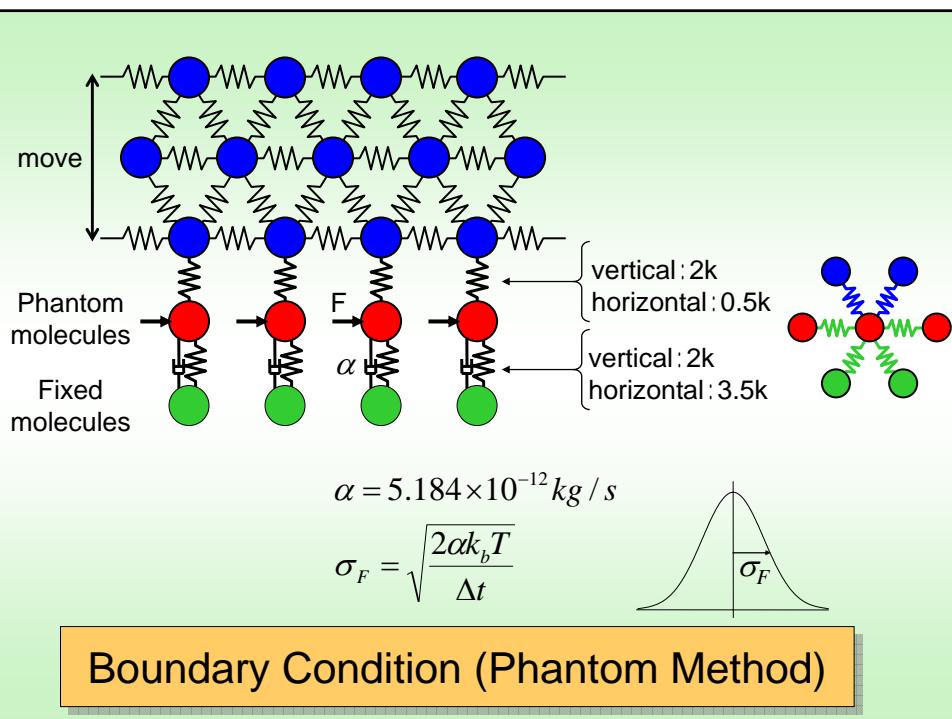


Boundary Condition (Solid Wall)

### One-Dimensional Function by Surface Integration



### Boundary Condition (Solid Wall)



### Boundary Condition (Phantom Method)

### Velocity Scaling

$$v_i' = v_i \sqrt{T_c/T}$$

Anderson method [Anderson (1980)]

Replace Velocity of Randomly Selected Molecule to Maxwell-Boltzmann Distribution

Nosé-Hoover Thermostat [Nosé(1984), Hoover(1985)]

$$m_i \frac{d^2\mathbf{r}_i}{dt^2} = \mathbf{F}_i - \zeta m_i \frac{d\mathbf{r}_i}{dt}$$

$$\frac{d\zeta}{dt} = \frac{2(E_k - E_k^0)}{Q}$$

### Temperature Control

Andersen (1980)

Change Box Size as if Piston is Connected

Parrinello and Rahman (1980, 1981)

Extension of Anderson: Change Shape of Box

Berendsen et al. (1984)

$$dP/dt = (P_c - P)/t_p$$

$$\mathbf{r}' = \chi^{1/3} \mathbf{r}$$

$$\chi = 1 - \beta_T \frac{\Delta t}{t_p} (P_c - P)$$

### Pressure & Stress Control

## Temperature

$$T = \frac{1}{3Nk_B} \left\langle \sum_{i=1}^N m_i v_i^2 \right\rangle \quad \text{For Monatomic Molecule}$$

Remember Thermodynamics

$$\frac{E_k}{n_f N} = \frac{1}{2} k_B T \quad \text{Kinetic energy for each freedom}$$

$$n_f = \begin{cases} 3 & \text{for monoatomic molecules} \\ 5 & \text{for diatomic molecules} \end{cases}$$

## Thermodynamics Properties

## Internal Energy or Total Energy

$$U = E_k + E_p = \frac{3}{2} N k_B T + \left\langle \sum_i \sum_{j>i} \phi(\mathbf{r}_{ij}) \right\rangle$$

Remember Thermodynamics for Ideal Gas

$$U = E_k = \frac{n_f}{2} N_A k_B T = \frac{n_f}{2} R_0 T \quad \text{per mol}$$

$$u = \frac{U}{m'} = \frac{n_f}{2} \frac{R_0}{m'} T = \frac{n_f}{2} RT \quad \text{per mass}$$

$k_B$  : Boltzmann Constant,  $1.38066 \times 10^{-23} \text{ J/K}$

$N_A$  : Avogadro Number,  $6.02205 \times 10^{23} \text{ 1/mol}$

$R_0$  : Universal Gas Constant,  $8.31433 \text{ J/(mol K)}$

$m'$  : Molecular Weight (kg/mol)

## Pressure by virial theorem

$$P = \frac{N}{V} k_B T - \frac{1}{3V} \left\langle \sum_i \sum_{j>i} \frac{\partial \phi}{\partial \mathbf{r}_{ij}} \cdot \mathbf{r}_{ij} \right\rangle$$

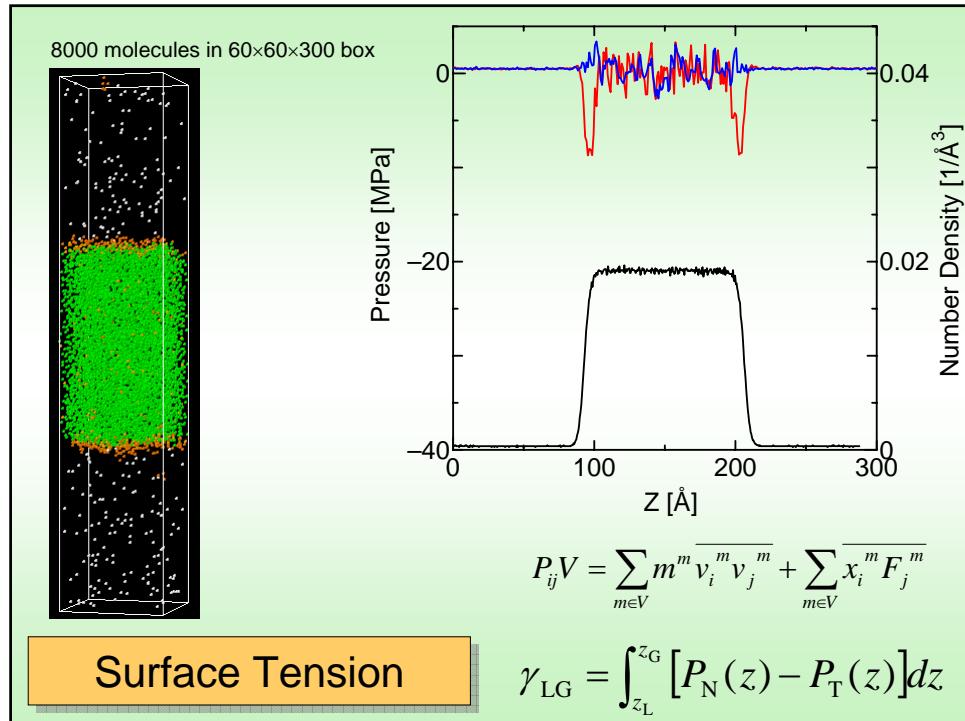
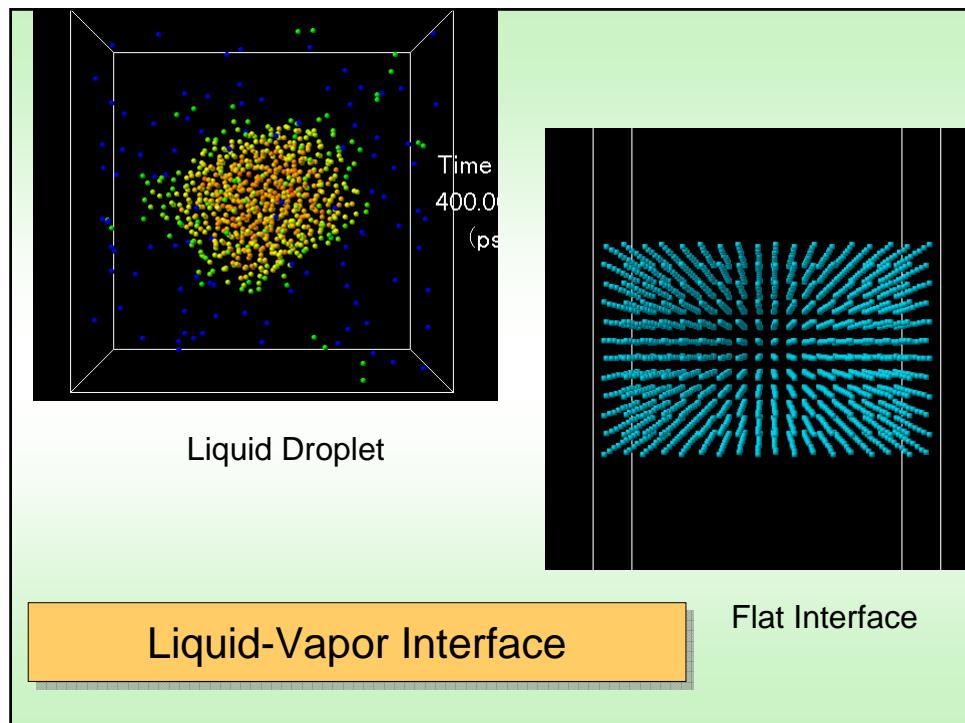
Thermodynamics for Ideal Gas

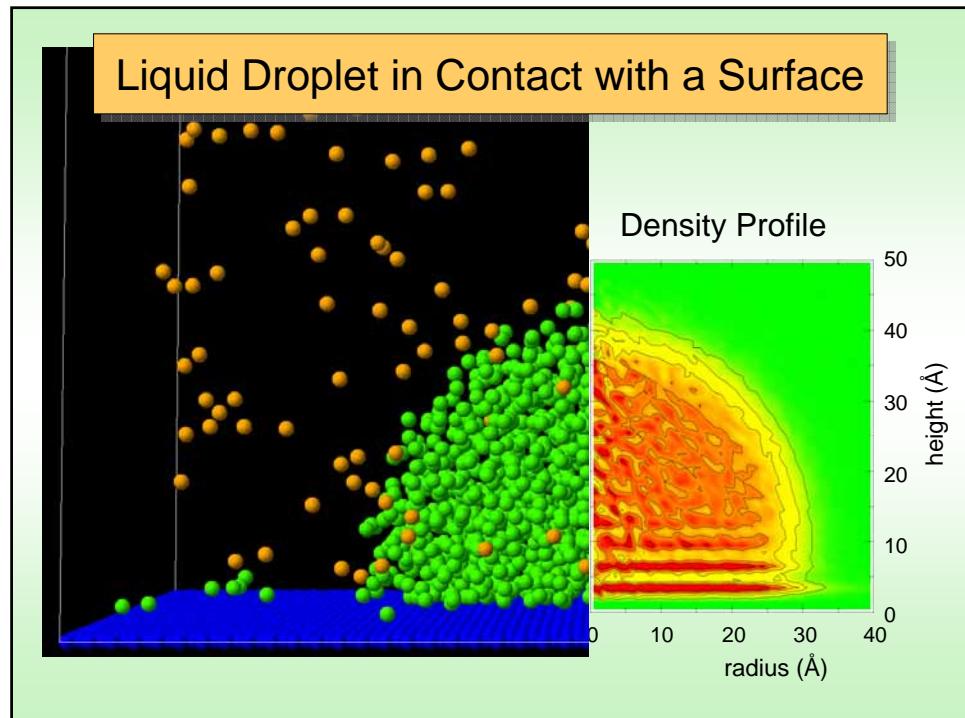
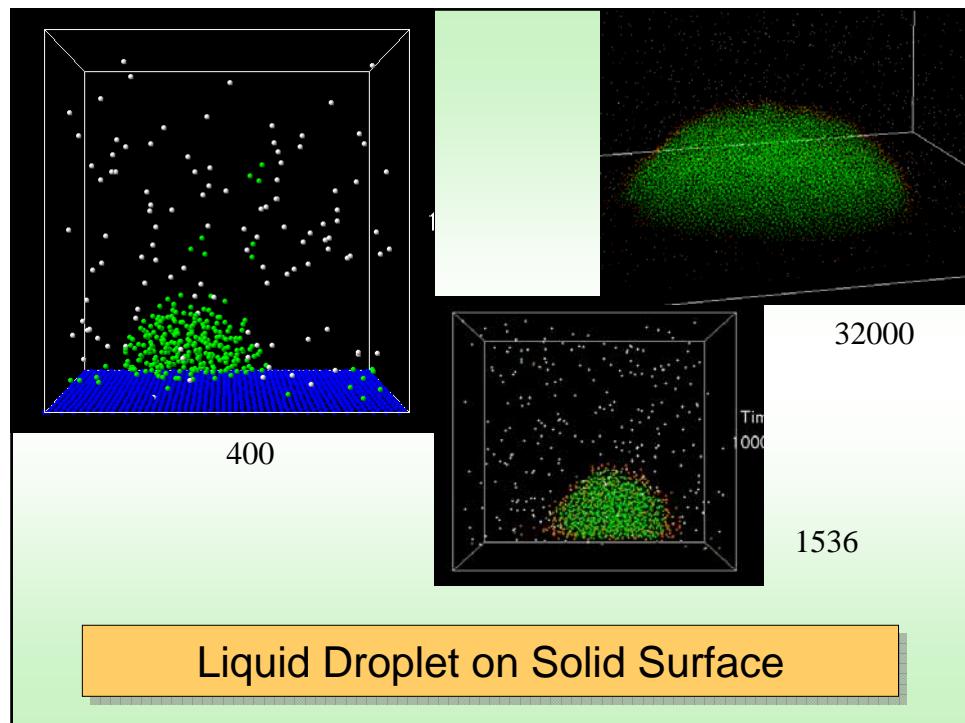
$$P = \frac{N}{V} k_B T$$

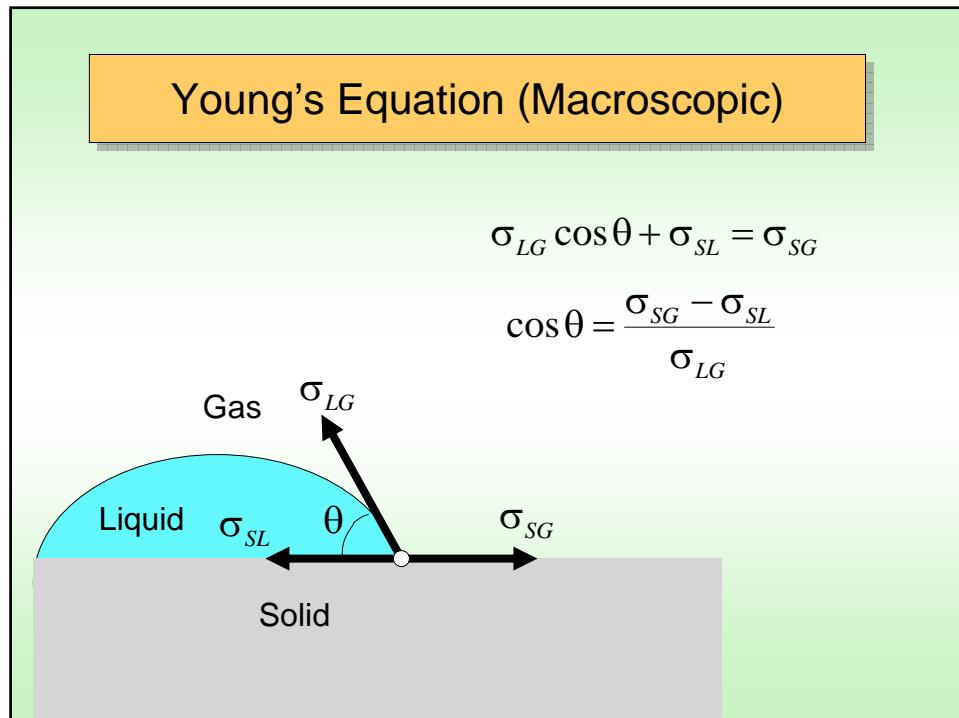
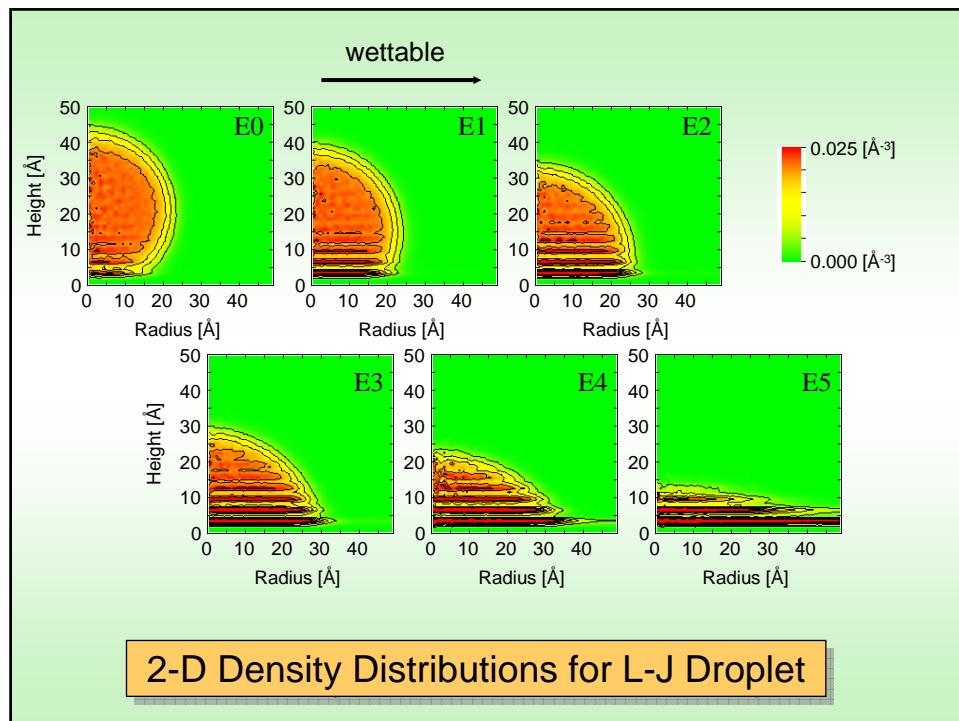
$$PV = n N_A k_B T = n R_0 T \quad \text{For n mol}$$

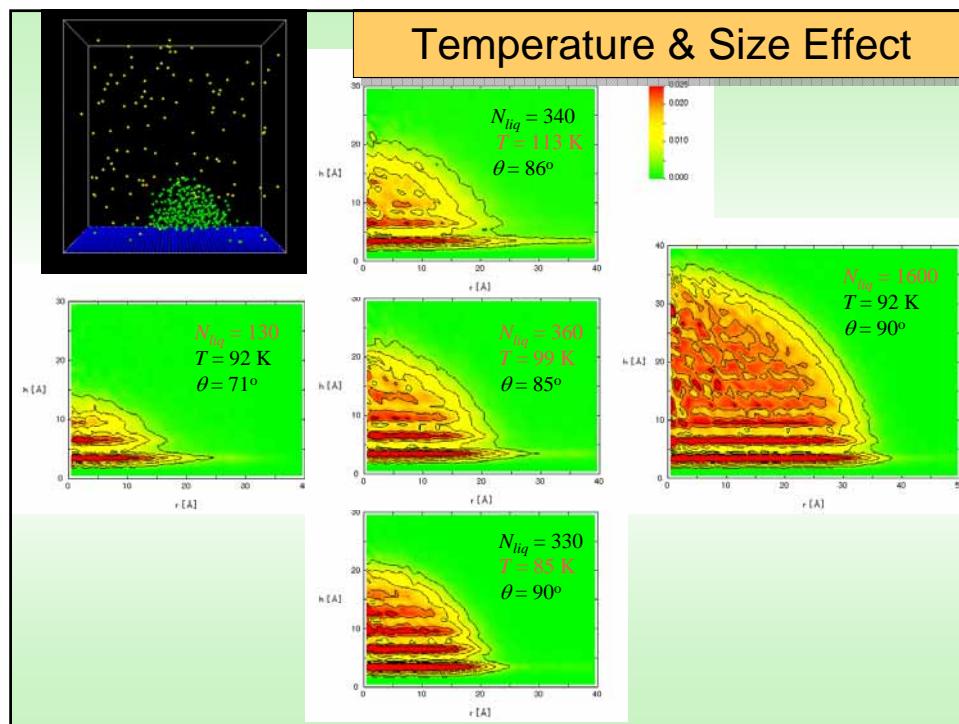
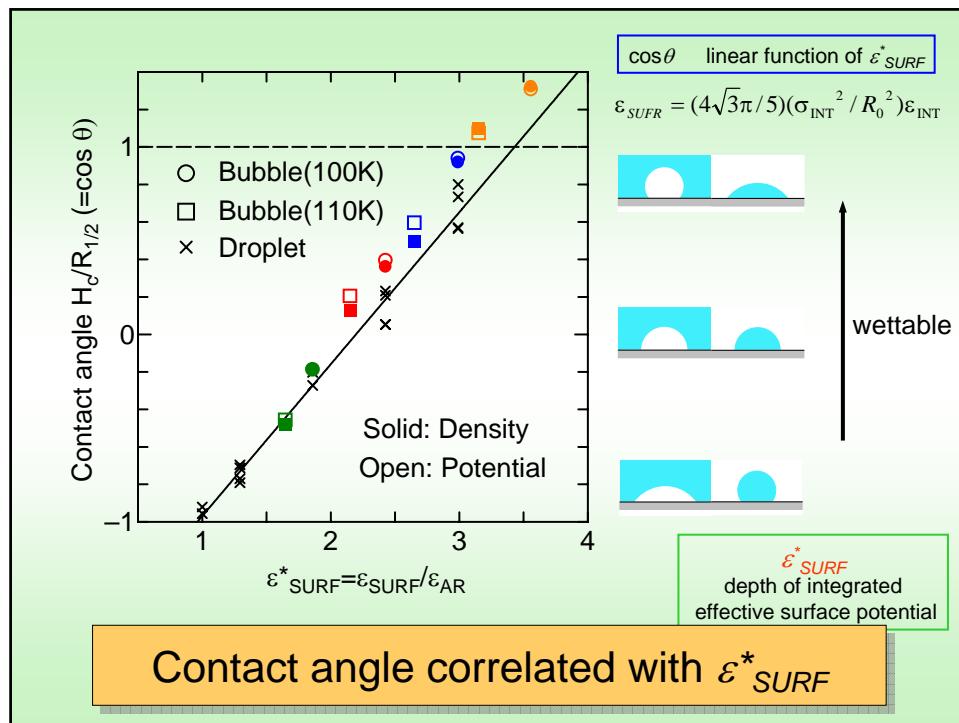
$$P = \frac{n N_A}{V} k_B T = \rho k_B T \quad \rho: \text{Number density}$$

相界面・濡れ・核生成

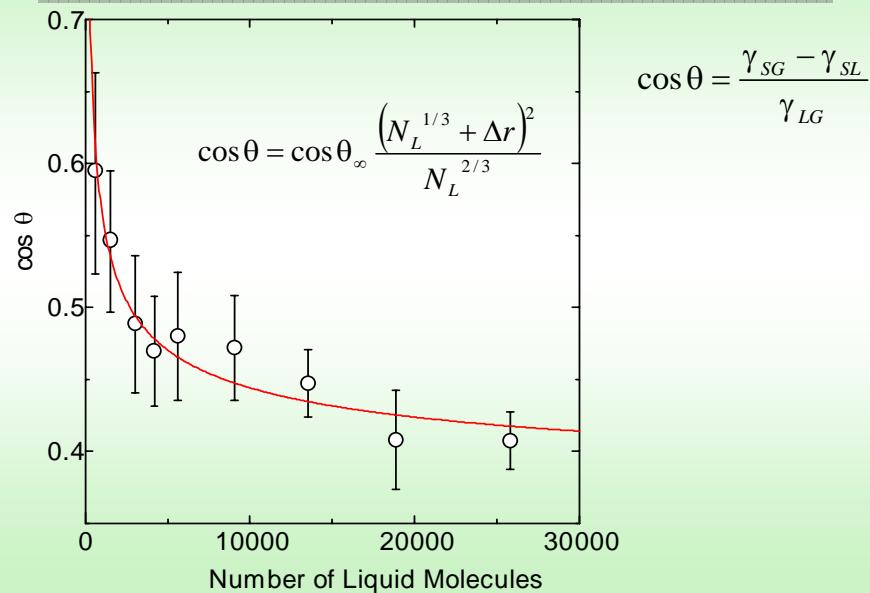




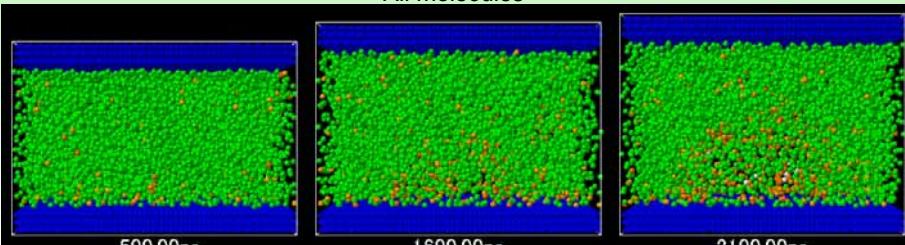




## Asymptotic Macro-System

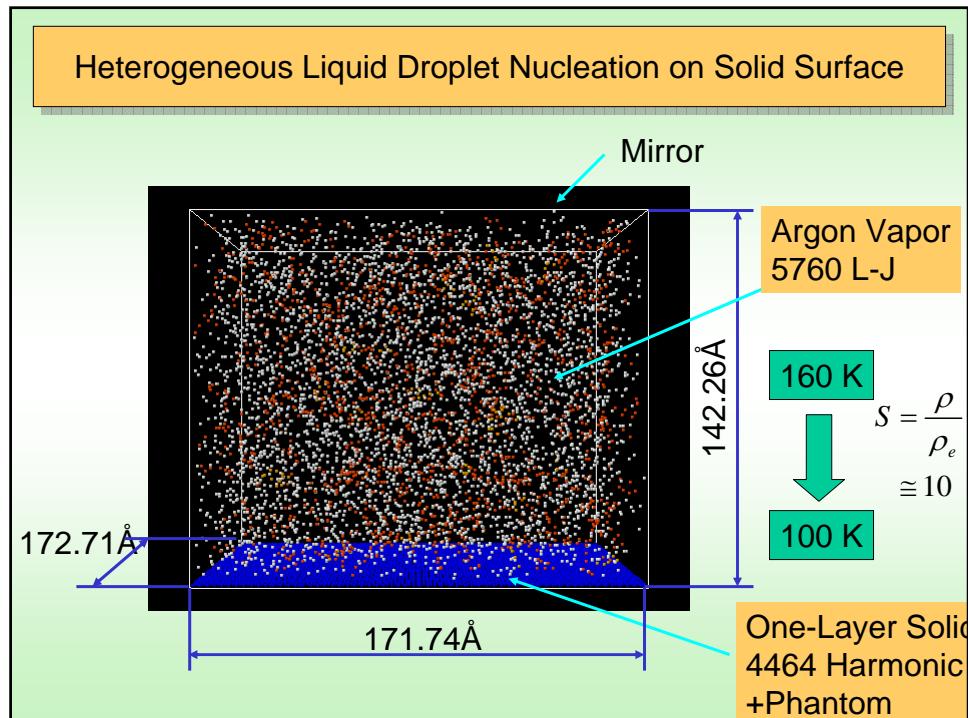
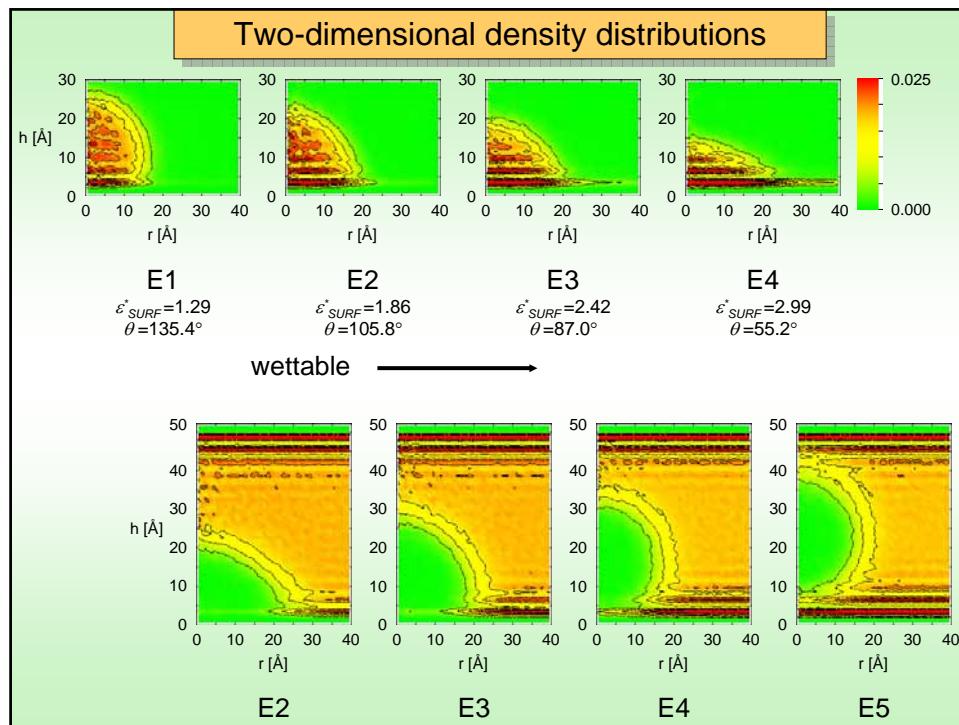


All molecules

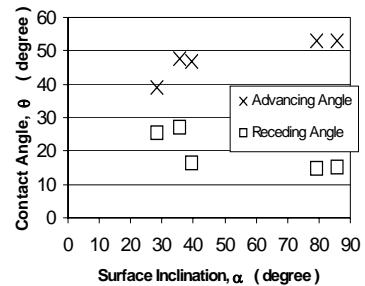
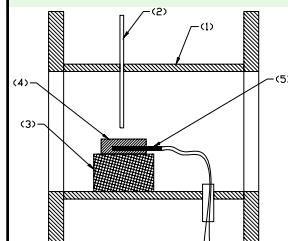


Sliced view (central 10Å)

Snapshots of bubble formation for E3



**Experiments by Satish G. KANDLIKAR**  
**Rochester Institute of Technology**

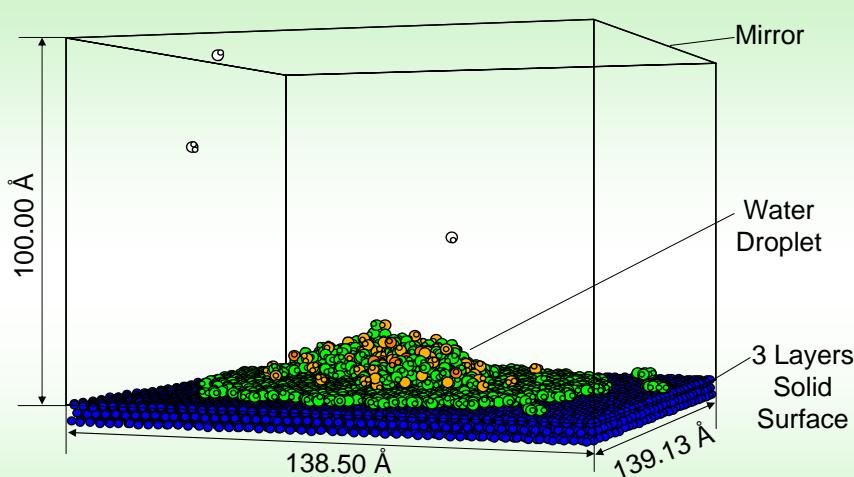


$m = 1.15 \times 10^{-6}$  kg,  $\alpha = 0^\circ$ ,  $T = 22^\circ\text{C}$ , and  $\theta = 22.05^\circ$ .

19.6 torr Vacuum, 18 MΩ de-ionized water

Surface roughness,  $R_a$ , value of  $0.02 \mu\text{m}$

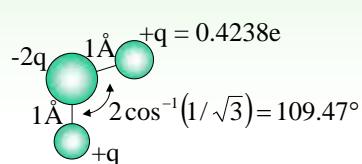
Modified RCA cleaning (1 part  $\text{NH}_4\text{OH}$ , 3 parts  $\text{H}_2\text{O}_2$ , and 15 parts  $\text{H}_2\text{O}$ )



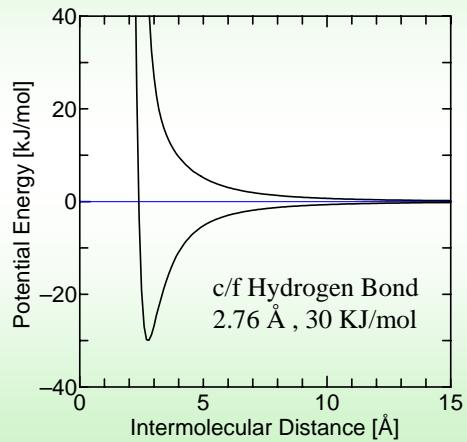
**System Configuration**  
**(water droplet on fcc(111) platinum surface)**

SPC/E H. J. C. Berendsen, et al. (1987)

$$\phi = 4\epsilon \left\{ \left( \frac{\sigma}{r_{OO}} \right)^{12} - \left( \frac{\sigma}{r_{OO}} \right)^6 \right\} + \sum_i \sum_j \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$



Cut-off Length  
25 Å

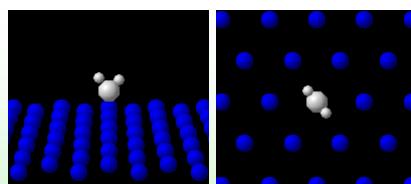
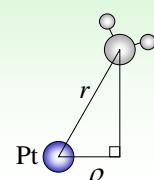


Water-Water Potential

E. Spohr & K. Heinzinger (1988)

$$\begin{aligned}\phi_{H_2O-Pt} &= \phi_{O-Pt}(r_{OPt}, \rho_{OPt}) + \phi_{H-Pt}(r_{H_Pt}) + \phi_{H_2Pt} \\ \phi_{O-Pt} &= [a_1 \exp(-b_1 r) - a_2 \exp(-b_2 r)] f(\rho) + a_3 \exp(-b_3 r) [1 - f(\rho)] \\ \phi_{H-Pt} &= a_4 \exp(-b_4 r) \\ f(\rho) &= \exp(-c\rho^2)\end{aligned}$$

$$\begin{aligned}a_1 &= 1.8942 \times 10^{-16} \text{ J}, & b_1 &= 1.1004 \text{ Å}^{-1} \\ a_2 &= 1.8863 \times 10^{-16} \text{ J}, & b_2 &= 1.0966 \text{ Å}^{-1} \\ a_3 &= 10^{-13} \text{ J}, & b_3 &= 5.3568 \text{ Å}^{-1} \\ a_4 &= 1.742 \times 10^{-19} \text{ J}, & b_4 &= 1.2777 \text{ Å}^{-1} \\ c &= 1.1004 \text{ Å}^{-1}\end{aligned}$$



Water-Platinum Potential (SH Potential)

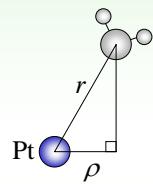
S.-B. Zhu and M. R. Philpott (1994)

$$\phi_{\text{H}_2\text{O-surf}} = \phi_{\text{H}_2\text{O-cond}} + \phi_{\text{an}}(\mathbf{r}_\text{O}) + \phi_{\text{isr}}(\mathbf{r}_\text{O}) + \sum_{\text{H}} [\phi_{\text{an}}(\mathbf{r}_\text{H}) + \phi_{\text{isr}}(\mathbf{r}_\text{H})]$$

$$\phi_{\text{H}_2\text{O-cond}} = \sum_{l,k} \frac{q_l q_k}{2 r_{lk}}$$

$$\phi_{\text{an}}(\mathbf{r}_p) = 4\epsilon_{p-\text{Pt}} \sum_j \left[ \left( \frac{\sigma_{p-\text{Pt}}^2}{(\alpha \rho_{pj})^2 + z_{pj}^2} \right)^6 - \left( \frac{\sigma_{p-\text{Pt}}^2}{(\rho_{pj}/\alpha)^2 + z_{pj}^2} \right)^3 \right]$$

$$\phi_{\text{isr}}(\mathbf{r}_p) = -4\epsilon_{p-\text{Pt}} \sum_j \frac{c_{p-\text{Pt}} \sigma_{p-\text{Pt}}^{10}}{r_{pj}^{10}}$$

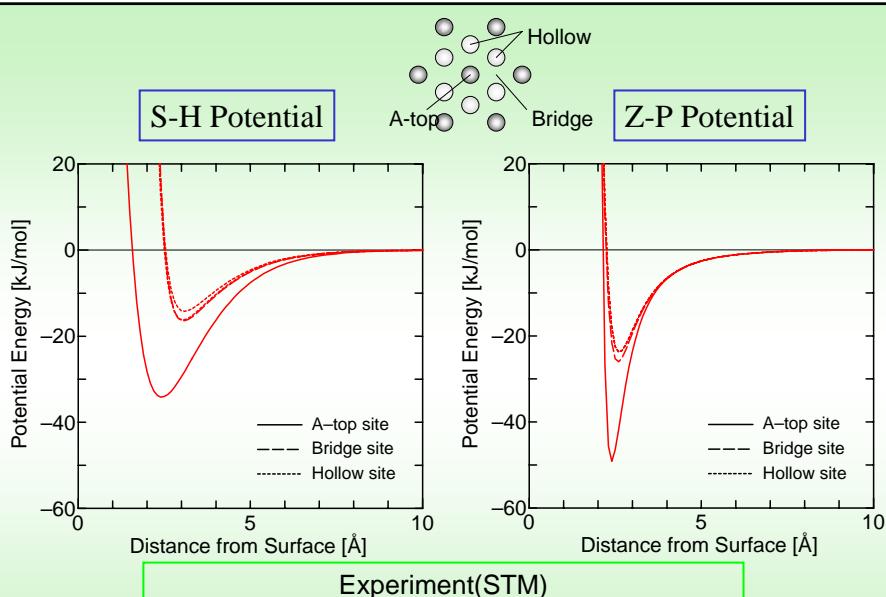


$$\alpha = 0.8$$

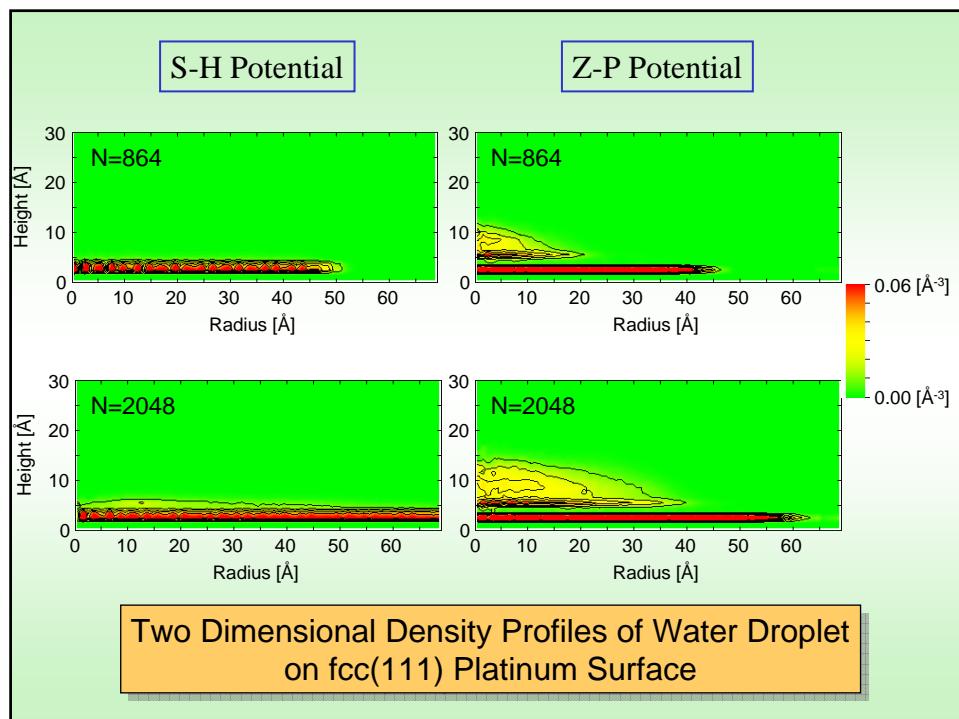
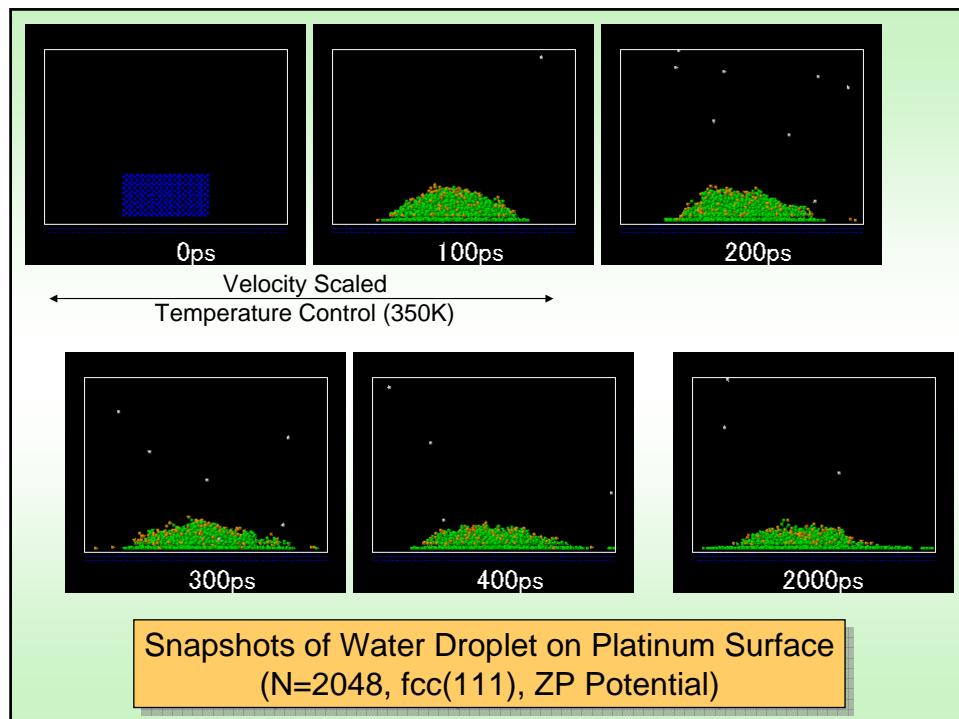
$$\sigma_{\text{O-Pt}} = 2.70 \text{ \AA}, \epsilon_{\text{O-Pt}} = 6.64 \times 10^{-21} \text{ J}, c_{\text{O-Pt}} = 1.28$$

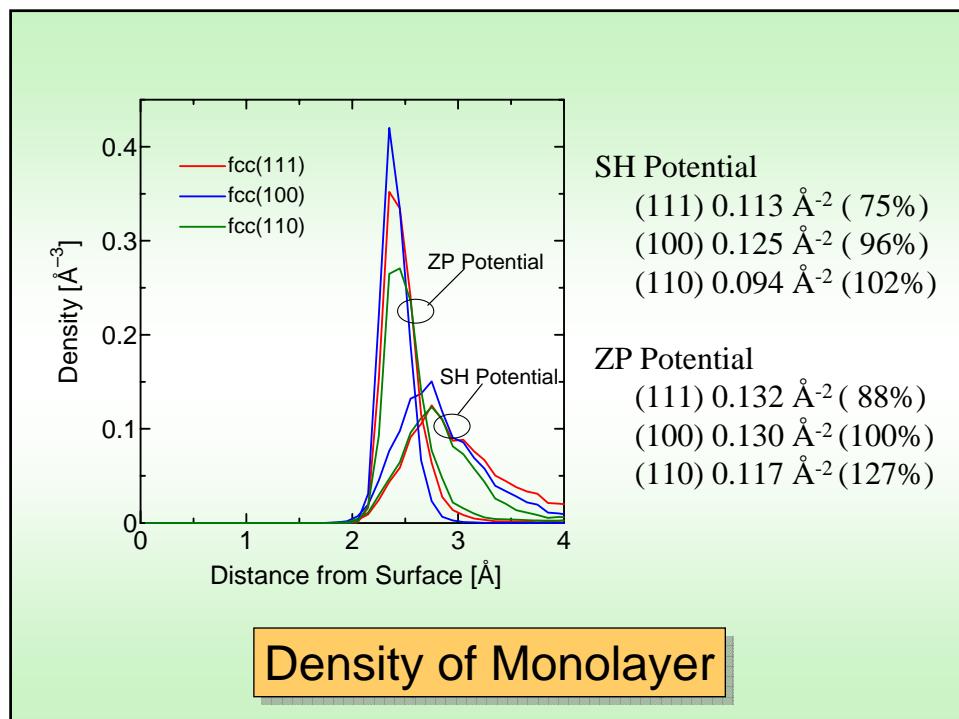
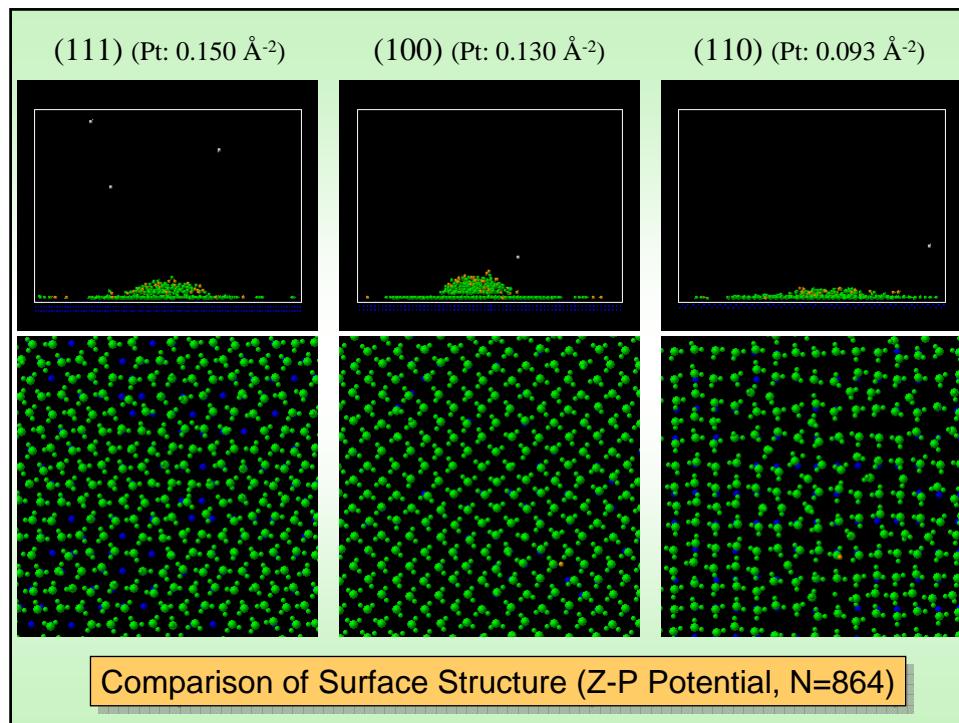
$$\sigma_{\text{H-Pt}} = 2.55 \text{ \AA}, \epsilon_{\text{H-Pt}} = 3.91 \times 10^{-21} \text{ J}, c_{\text{H-Pt}} = 1.2$$

### Water-Platinum Potential (ZP Potential)

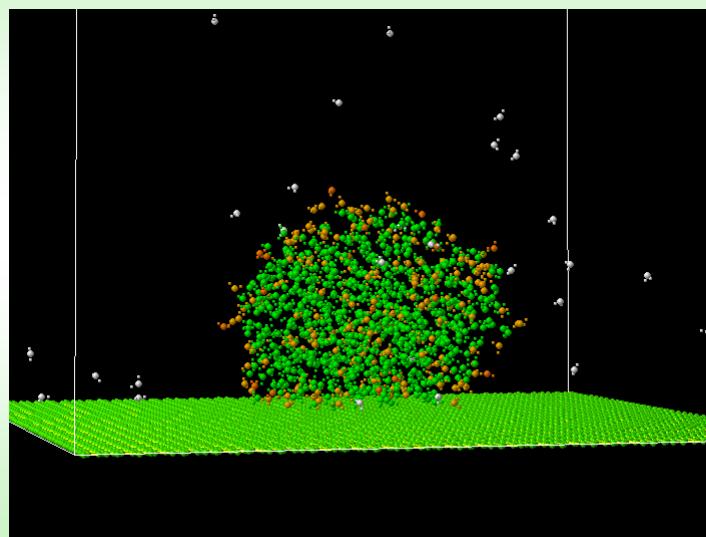


### Comparison of Water-Platinum Potential

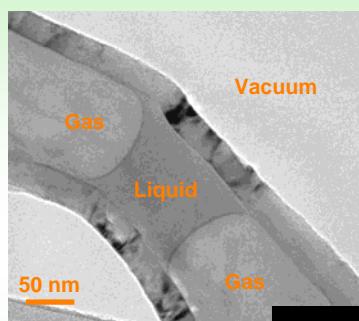




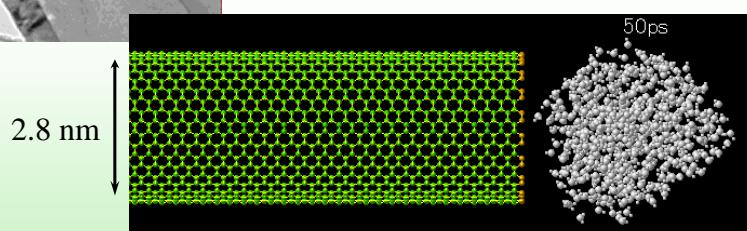
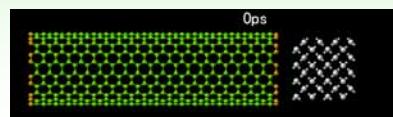
## Water Droplet on Graphite



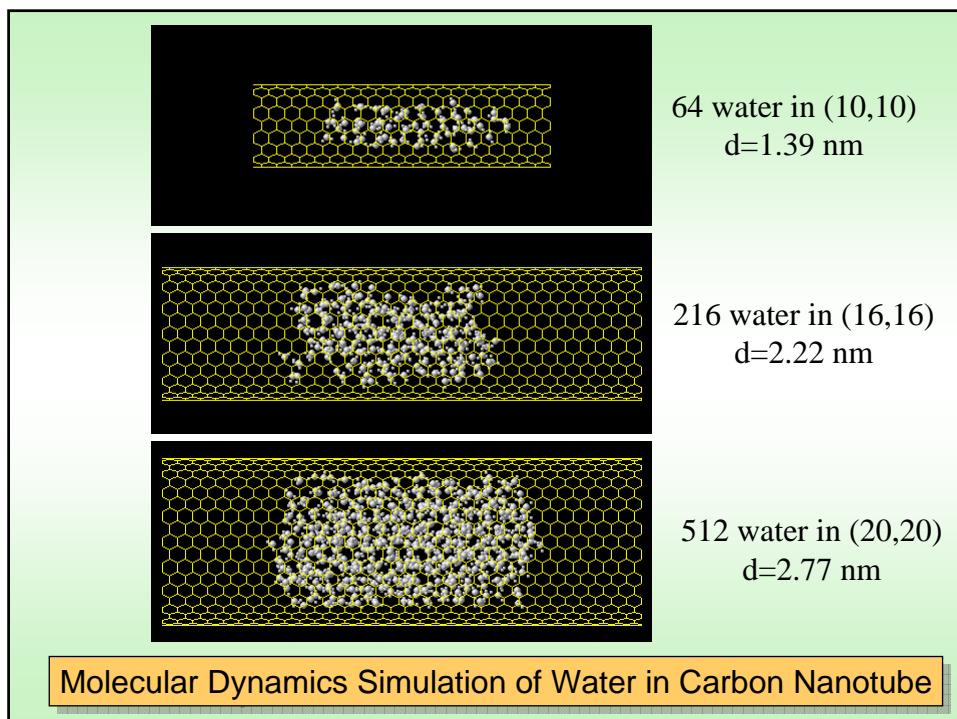
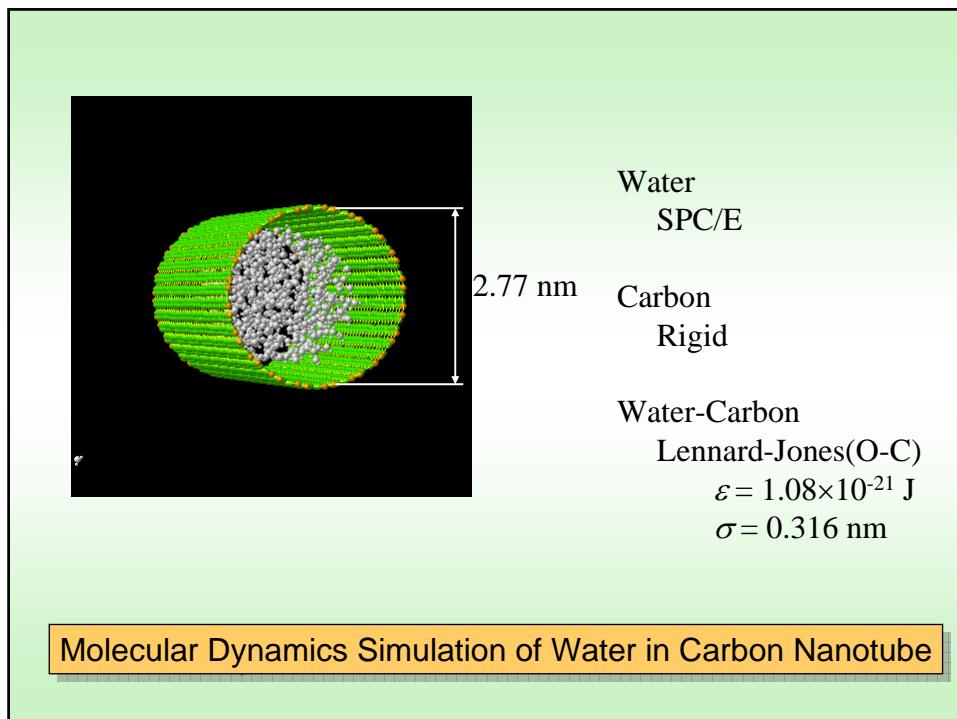
## Liquid in Carbon Nanotube



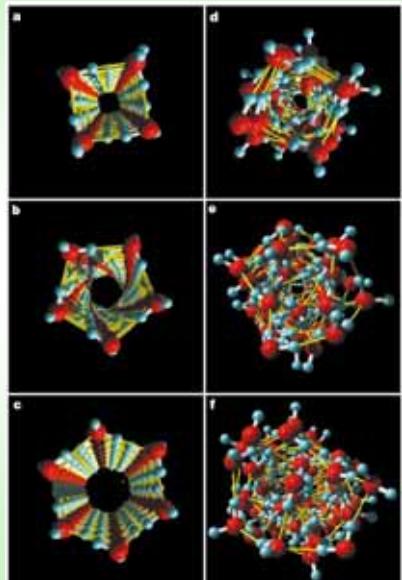
Y. Gogotsi et al., APL (2001).



T. Kimura & S. Maruyama (2002).



### Molecular Dynamics Simulation of Ice Water in Carbon Nanotube



K. Koga, et al., *Nature*, 2001

(14,14) D=1.11 nm ?

(15,15) D=1.19 nm ?

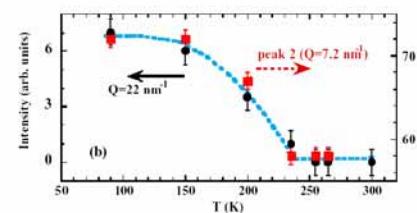
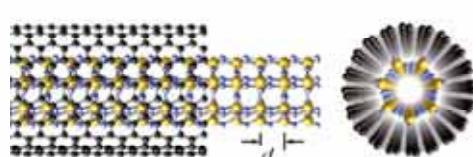
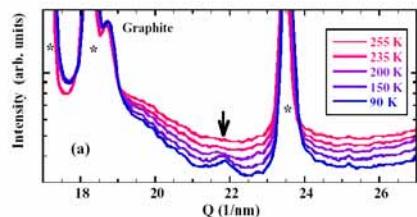
(16,16) D=1.26 nm ?

50~500 MPa

### Ice Water in Carbon Nanotube

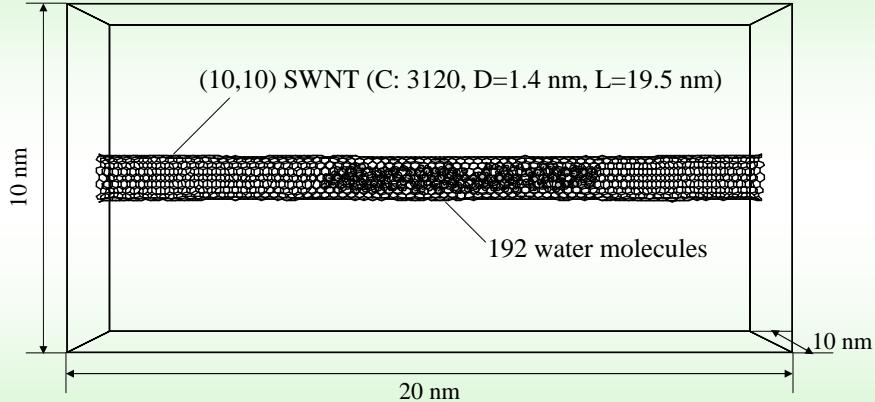
“Phase Transition in Confined Water Inside Carbon Nanotubes”  
Y. Maniwa, et al., *J. Phys. Soc. Jpn.*, 2002.

X-ray diffraction  
water adsorption in single-walled carbon nanotube



phase transition at 235K

## System Configuration



Water: SPC/E  
Carbon: Brenner

Total Energy  $E_b$ :

$$E_b = \sum_i \sum_{j(i)} \{V_R(r_{ij}) - B_{ij}^* V_A(r_{ij})\}$$

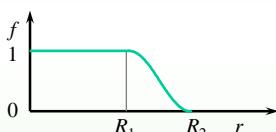
## C-C Potential Function

$$V_R(r) = f(r) \frac{D_e}{S-1} \exp\{-\beta \sqrt{2S}(r-R_e)\} \quad V_A(r) = f(r) \frac{D_e S}{S-1} \exp\left\{-\beta \sqrt{\frac{2}{S}}(r-R_e)\right\}$$

$$B_{ij}^* = \frac{B_{ij} + B_{ji}}{2}, \quad B_{ij} = \left[ 1 + \sum_{k \neq i,j} \{G_c(\theta_{ijk}) f(r_{ik})\} \right]^{-\delta}$$

Cut-off function

$$G_c(\theta) = a_0 \left( 1 + \frac{c_0^2}{d_0^2} - \frac{c_0^2}{d_0^2 + (1 + \cos \theta)^2} \right)$$



Potential parameters

$D_e = 6.325 \text{ eV}$	$S = 1.29$	$\beta = 1.5 \text{ \AA}^{-1}$	$R_e = 1.315 \text{ \AA}$
$\delta = 0.80469$	$a_0 = 0.011304$	$c_0 = 19$	$d_0 = 2.5$
$R_1 = 1.7 \text{ \AA}$	$R_2 = 2.0 \text{ \AA}$		

D. W. Brenner, *Phys. Rev. B*, **42** (1990) 9458.

## Water-Carbon Potential

Lennard-Jones interaction

$$\phi(r) = 4\epsilon_{\text{CO}} \left[ \left( \frac{\sigma_{\text{CO}}}{r_{\text{CO}}} \right)^{12} - \left( \frac{\sigma_{\text{CO}}}{r_{\text{CO}}} \right)^6 \right]$$

$\epsilon_{\text{CO}} = 0.108 \times 10^{-21} \text{J}$   
 $\sigma_{\text{CO}} = 3.19 \times 10^{-10} \text{m}$

Quadrupole interaction(carbon atoms and partial charges on water)

$$\phi(r) = \frac{1}{3} \frac{q}{4\pi\epsilon_0} \sum_{\alpha,\beta} \Theta_{\alpha,\beta} \frac{3r_\alpha r_\beta - r^2 \delta_{\alpha\beta}}{r^5}$$

$$-2\Theta_{x'x'} = -2\Theta_{y'y'} = \Theta_{z'z'} = -3.03 \times 10^{-40} \text{C}$$

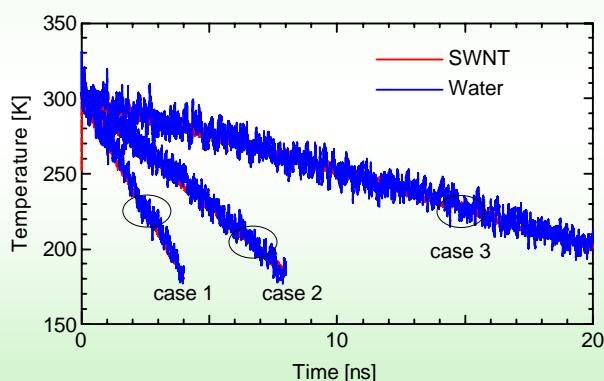
Walther, J. H., et al. (2001)

## Time Variations of Temperature

PV

50 psまで: 系全体に温度制御(300 K)

50 ps以降: SWNTのみ温度制御(一定熱流速)



case 1

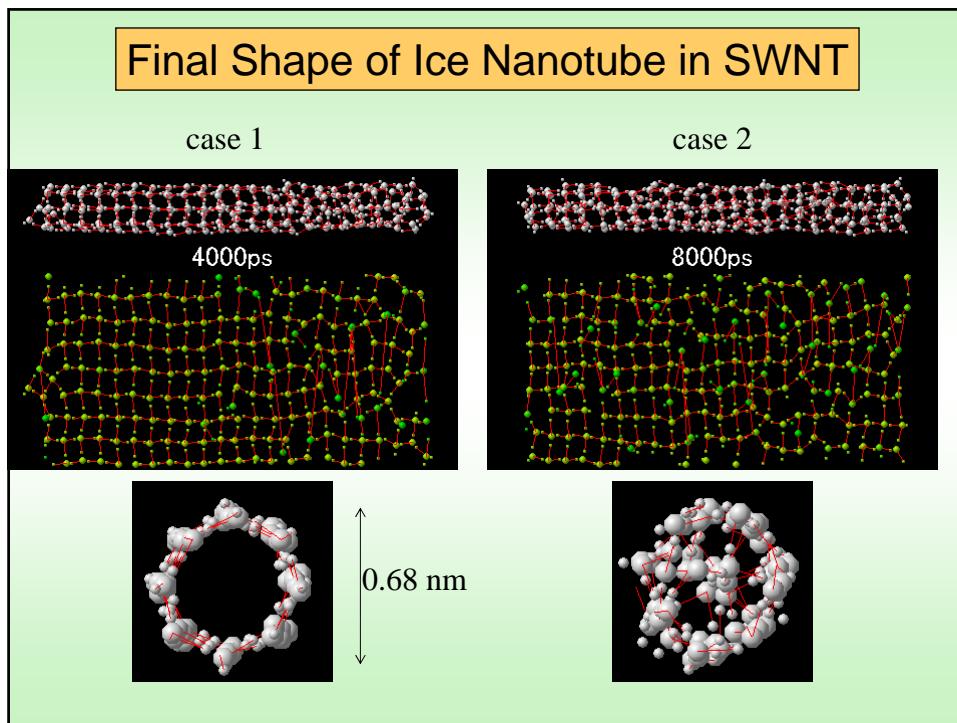
$$Q = -5.0 \times 10^{-9} \text{ W}$$

case 1

$$Q = -2.5 \times 10^{-9} \text{ W}$$

case 3

$$Q = -1.0 \times 10^{-9} \text{ W}$$



### Critical Radius & Free Energy

$$\text{Area } A = 4\pi r^2 \quad \text{Volume } V = \frac{4}{3}\pi r^3$$

Surface tension  $\gamma$

Free Energy of Cluster  $\Delta G = \gamma A + \Delta g V$

Free energy difference  
In solid and vapor for unit volume  $\Delta g = -\rho_l k_B T \ln(\frac{\rho}{\rho_e})$   
Assumption of ideal gas

Number of molecules $n = \rho_l V$	Supersaturation Ratio $S = \frac{\rho}{\rho_e}$
$r^* = \frac{2\gamma}{\rho_l k_B T \ln S}$	$n^* = \frac{32\pi\gamma^3 f}{3\rho_l^2 (k_B T \ln S)^3}$
$\Delta G^* = \frac{16\pi\gamma^3 f}{3(\rho_l k_B T \ln S)^2}$	

## Critical Radius & Free Energy

$$\text{Area } A = 4\pi r^2$$

$$\text{Volume } V = \frac{4}{3}\pi r^3$$

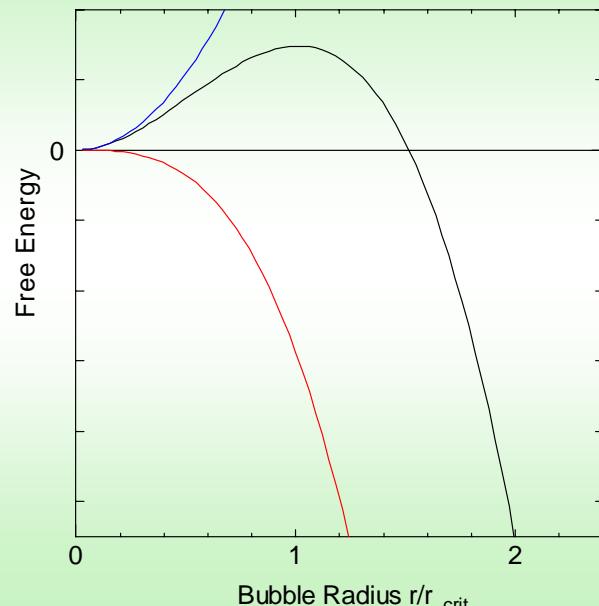
Surface tension  $\gamma$

$$\text{Free Energy of Cluster} = \gamma A + \Delta g V + \Delta G_L$$

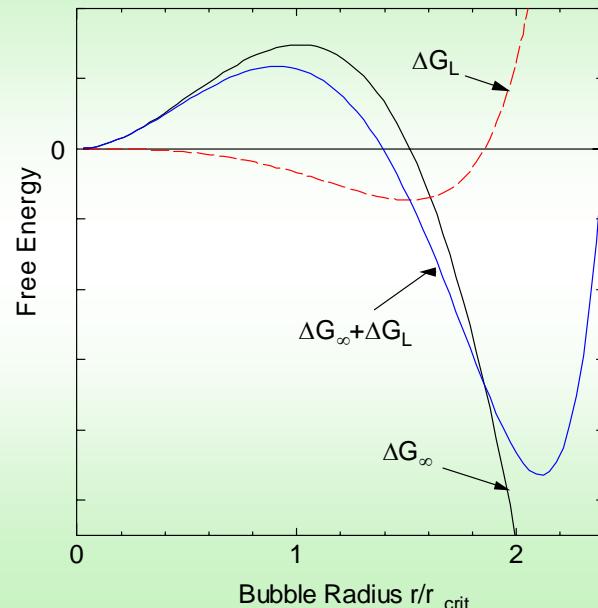
$$\text{Free Energy Gain by } \Delta g_L \left( L^3 - \frac{3}{4}\pi r^3 \right)$$

$$\Delta g_L = f\left(\frac{\rho_L}{\rho_{sat}}\right) = f\left(\frac{V_{sat}}{V_L}\right) = f\left(\frac{L^3 - \frac{4}{3}\pi r_{sat}^3}{L^3 - \frac{4}{3}\pi r^3}\right)$$

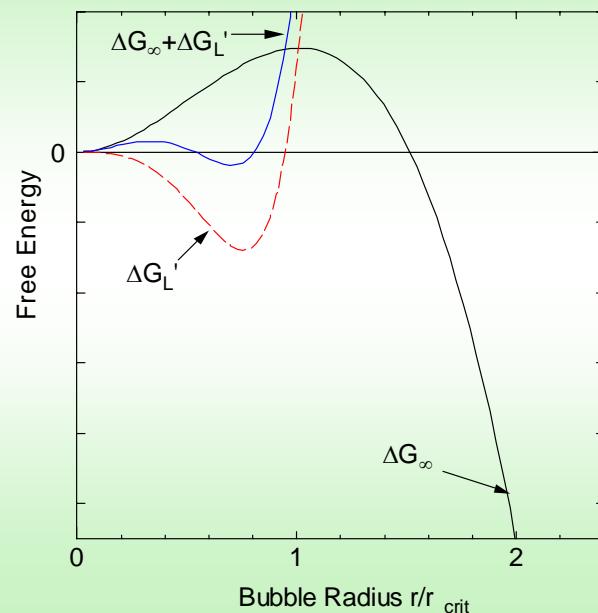
## Note on Stability of Nano-Bubble



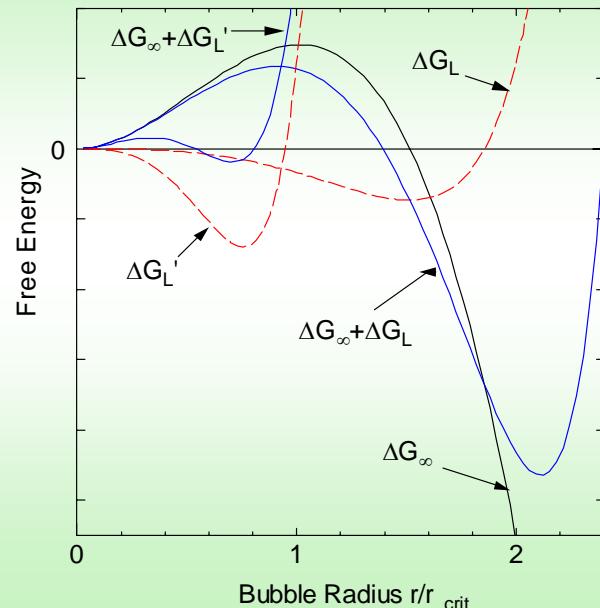
### Note on Stability of Nano-Bubble



### Note on Stability of Nano-Bubble



## Note on Stability of Nano-Bubble



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