For a substance expressed with Lennard-Jones (12-6) potential

\[
\phi(r) = 4\varepsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \right], \quad (1)
\]

answer the following questions.

(a) Derive the non-dimensional forms for following variables.
- Temperature \( T \)
- Force \( F \)
- Pressure \( P \)
- Surface tension \( \gamma \)
- Thermal conductivity \( \lambda \)

(b) Calculate the pair separation at which the Lennard-Jones potential is a minimum.

(c) Guess why “4” is used in equation (1). Isn’t it simpler to define the potential as

\[
\phi(r) = \varepsilon \left[ \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \right].
\]

(d) Use the Newton’s second law to show that in terms of the potential parameters \( \varepsilon \) and \( \sigma \), the unit of time is \( \sigma\sqrt{m/\varepsilon} \), where \( m \) is the mass of one atom.

(e) The long-range correction for potential energy \( E_p \) is expressed as

\[
\frac{E_p}{N} = \frac{\bar{E}_p}{N} + E_{pLR}
\]

\[
E_{pLR} = 2\pi \rho \int \phi(r)r^2 dr
\]

Derive the following expression for Lennard-Jones potential.

\[
E_{pLR}^* = \frac{8\pi \rho^*}{3(r_c^*)^3} \left( \frac{1}{3(r_c^*)^6} - 1 \right)
\]

(f) Consider two molecules at \( r_i \) and \( r_j \). Probe that the force \( \mathbf{F}_i = -\nabla_i \phi \) acting on molecule \( i \) is expressed as

\[
\mathbf{F}_i = 24\varepsilon \left[ 2 \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^{6} \right] \frac{\mathbf{r}_{ij}}{r_{ij}^2} \quad \text{where} \quad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j \quad \text{and} \quad r_{ij} = |\mathbf{r}_{ij}|.
\]