## Shigeo Maruyama The University of Tokyo

Let's assume that number density of citation N(t) decays by exponential function as

$$N(t) = A \exp(-\frac{t}{\tau}) \tag{1}$$

Here, number of citations during t $\sim$ t+dt is N(t)dt. Prefactor A and decay time  $\tau$  are specific to each paper. Then, the expected life-time number of citations C<sub>fin</sub> is expressed as

$$C_{fin} = \int_0^\infty N dt = A \int_0^\infty \exp(-\frac{t}{\tau}) dt$$
$$C_{fin} = A \int_0^\infty \exp(-\frac{t}{\tau}) dt = A(-\tau) \left[ \exp(-\frac{t}{\tau}) \right]_0^\infty = A \tau \qquad (2)$$

On the other hand, number of citations  $C_0$  at time  $t_0$  is

$$C_{0} = A \int_{0}^{t_{0}} \exp(-\frac{t}{\tau}) dt = A(-\tau) \left[ \exp(-\frac{t}{\tau}) \right]_{0}^{t_{0}} = A(-\tau) \left( \exp(-\frac{t_{0}}{\tau}) - 1 \right)$$
$$= C_{fin} \left( 1 - \exp(-\frac{t_{0}}{\tau}) \right)$$

Hence,  $C_{fin} = \frac{C_0}{1 - \exp(-\frac{t_0}{\tau})}$  (3)

By using equation (3),  $C_{fin}$  can be calculated from  $C_0$  and  $t_0$  assuming the decay time  $\tau$ .

Here, half-life time  $\tau_{half}$  is related to  $\tau$  as

$$\exp(-\frac{\tau_{half}}{\tau}) = \frac{1}{2} \text{ or } -\frac{\tau_{half}}{\tau} = \log\left(\frac{1}{2}\right)$$
  
Hence,  $\tau = -\frac{\tau_{half}}{\log(0.5)} \approx 1.44\tau_{half}$ 

Half-life time listed in Journal Citation Report typically varies from 4 to 10 years depending on a journal. Since most of traditional journals has this half-life time  $\tau_{half}$  about 7, let's assume  $\tau$  as 10 years.

Then, equation (3) becomes 
$$C_{fin} = \frac{C_0}{1 - \exp\left(-\frac{t_0 [month]}{120[month]}\right)}$$